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Ancient Mathematics and the Development of Primitive Culture

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THE HISTORIAN has been unable to ascertain whether the number sense was a part of the original endowment of the *genus homo* or not. It is known, however, that this innate or acquired ability possessed by man in the early periods of his development permitted him to recognize that a change had been made in a small group of objects when one had been deleted or added.

Counting came much later than the number sense and was developed much more rapidly by some people than by others. This evolution of counting was followed by number words and number symbols. Like counting, number systems evolved much earlier in some places than in others. An examination of the histories of uncivilized peoples reveals the absence of a workable number system. By a usable or workable number system is meant a system of number words and number symbols sufficient to permit the development of elementary mathematics. Anthropology cites no instance of an advanced social culture preceding a workable number system.

The Ahmes Papyrus evidences the fact that the Egyptians had a well developed number system nearly two millennia before the beginning of our era. This number system made possible the development of units of weights and measures which must of necessity precede trade and industry. With commerce came the propagation of ideas and learning. Without an increase and dissemination of knowledge a civilization could not have evolved in the fertile valley of the life-giving Nile.

It was natural for the measurement of area to follow on the heels of linear measurement. Likewise, a study of three dimensional geometry was naturally begun before the elements of plane geometry were well developed. Prior to the inception of the Ahmes Papyrus, Egyptian mathematicians were able to determine the areas of rectangles, triangles, and circles, and the volumes of cylinders and prisms, and they knew that in a right triangle the relation of the length of two sides determines one of the angles. The author of the papyrus took up problems of volume, then problems of area, and finally problems involving relative length of the sides of a triangle.[1] The crowning achievement of ancient Egyptian mathematicians is the so-called formula for the volume of the frus-

[1] Arnold Chase and Henry Manning, *The Rhind Mathematical Papyrus*, Oberlin, Ohio: The Mathematical Association of America, 1927, I, 35.

tum of a square pyramid.[2] Thus nearly four thousand years ago the development of Egyptian geometry had been sufficient to permit the measurement and division of grain and other produce. It made possible surveying and the division of land among the people and a redivision after the swollen Nile had receded. Aristotle said that mathematics had its birth in Egypt, because there the priestly class had the leisure needful for its study.[3] Regardless of what claims may truthfully be made for the antiquity of mathematics in various other countries, claims of even greater validity can justly be made for the science in the Nile valley.[4]

Here mathematics made it practicable for people to dwell near each other in peace and harmony. An elementary knowledge of this useful science made it possible for the Egyptian farmer to return after the annual flood to an equal amount of more fertile land. Had a knowledge of surveying not been developed, the return following the recession of the overflow would probably have been accompanied by strife and bloodshed.

Northeastward from the Nile valley, at a distance of more than a thousand kilometers, was located ancient Babylon. In and around this city lived the Sumerians, who evolved one of the earliest known civilizations. Like the Egyptians they inhabited a fertile valley where food could be grown in abundance. They developed a number system and the elements of mathematics four thousand years ago and with this came the evolution of commerce and industry.

The Babylonians knew that the area of a rectangle was equal to the product of two adjacent sides and that the area of a right triangle could be found by taking one half the product of the length of the sides about the right angle. They were acquainted with the so-called Pythagorean theorem and also with the fact that the angle in a semicircle is a right angle.[5] These early Sumerians had tables of square root and tables of cube root, and they understood equations as well as arithmetic and geometric progression.[6] Numerous problems prove that the Babylonian mathematicians were familiar with our formula for the solution of a quadratic equation twenty centuries before the dawn of the Christian era.[7] They knew how to dig canals, measure grain, calculate interest for loans of silver, and numerous other problems pertaining to business and industry.

The people of the Tigris-Euphrates valley had a system of numbers that made possible the development of simple arithmetic operations. These simple operations were followed by more advanced and more useful mathematics—making trade and industry practicable. Their knowledge of geometry made feasible the division of real estate which was necessary to peace and harmony at home. The arithmetic of ancient Babylon gives evidence to the fact that these people traded and exchanged goods. Industry at home was followed by commerce and foreign trade. Local trade stimulated the dissemination of knowledge at home and the exchange of goods with foreign people was accompanied by an exchange of ideas and learning. This would have been impossible without a knowledge of mathematics which had been preceded by usable number symbols and number words.

A brief examination of the Egyptian and Babylonian number systems and a comparison of their symbols with our modern

[2] G. A. Miller, "A Wide-Spread Error Relating to Egyptian Mathematics," *Science*, LXXXI, 1935, 152.

[3] Florian Cajori, *A History of Mathematics*, New York: The Macmillan Co., 1931, p. 9.

[4] David Eugene Smith, *History of Mathematics*, New York: Ginn and Company, 1923, I, 41.

[5] R. C. Archibald, "Babylonian Mathematics," *Science*, LXX, 1929, 67.

[6] "Two Babylonian Multiplication Tables in Ontario," *Art and Archaeology*, XXVI, 1928, 145.

[7] R. C. Archibald, "Babylonian Mathematics with Special Reference on Recent Discoveries," THE MATHEMATICS TEACHER, XXIX, 1936, 217.

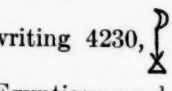
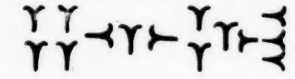
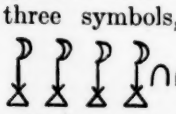
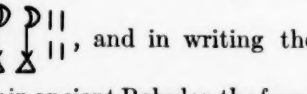
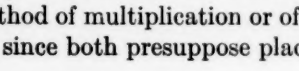
symbols should be of interest. The basic figure of the Babylonian system was the wedge. A lone wedge standing on its point, , respresented 1. Nine wedges properly arranged and of the right magnitude, , represented 9. Similarly, 80 was represented by eight horizontal wedges pointing to the left, , 10 being represented by one such wedge, . The verticle wedge standing on its point followed by a horizontal wedge pointing to the right, , represented 100. With the exception of the distinct symbol for 10, the additive principle was used in writing symbols up to 100. In writing 400 the Babylonians did not write 100 four times, but wrote it as four times 100, . The multiplicative principle was also used in writing 1000, , (10 times 100) and figures of higher denomination. To represent 1000 required only three wedges in ancient Babylon, which is one less than the number of symbols required to represent the same in Hindu-Arabic notation. To indicate five times one thousand or five thousand, , required a number of wedges equal to twice the number of symbols necessary to represent the same in Hindu-Arabic. To write 8976, the Babylonian scribe wrote symbols representing the following numbers in the following order: 8, 1000, 9, 100, 10 (seven times), and 1 (six times). If each wedge is considered a character, the number of characters used was $8+3+9+2+7+6=35$. If the scribe wrote three characters per second, he could write five such numbers in one minute. At the same rate of writing characters, forty-five such numbers can be written by using the Hindu-Arabic symbols. For this specific example, our modern symbols require approximately one ninth as much time for writing. Consider 564, which is more of an average number, since the mean of the individual figures is 5. In writing this number $5+2+6+4=17$ characters are used. At the rate designated above, it would have required nearly six seconds to write this figure in ancient Babylon, and one second to represent the same today.

The Egyptian system resembled the Babylonian system in that it employed the additive principle and differed in that it did not employ the multiplicative principle. Nine was represented by nine strokes, , each stroke representing 1. Ahmes used nine inverted U's, , in writing 90, each character representing 10. Ten to any integral positive power was represented by a distinct symbol unless the number was beyond the system.[8] One hundred was represented by the coil of a rope, ; 1,000 by a lotus flower, ; 10,000 by an upright bent finger, ; 100,000 by a tadpole, ; 1,000,000 by a god with uplifted hands, .[9] Therefore 1,984,567 would have been written: a god, then nine tadpoles, followed by eight bent fingers, followed by four lotus flowers, followed by five rope coils, followed by six inverted U's, followed by seven strokes. Thus the number of Egyptian characters necessary to represent the above number was $1+9+8+4+5+6+7=40$. The number of characters necessary to represent 8976 was $8+9+7+6=30$. In representing any number by Egyptian symbols, the actual number of characters required is equal to the sum of the individual figures. Thus if 564 is again used as an average number, five times as many

[8] Moritz Cantor, *Vorlesungen über Geschichte der Mathematik*, Leipzig: Verlag und Druck von B. F. Teubner, 1922, I, 82.

[9] R. C. Archibald, "Mathematics Before the Greeks," *Science*, LXXI, 1930, 110.

symbols are required as is the case when the Hindu-Arabic notation is used. Assuming that Ahmes could make an Egyptian symbol in the same time required for a modern mathematician to write a modern symbol, his speed in writing numbers was approximately one fifth that of the American mathematician.

The absence of a zero was another deficiency in the early number systems. In writing 4230,  , the Egyptians used four symbols, each representing 1,000, two symbols, each representing 100, and three symbols, each representing 10. In ancient Babylon 4230 was written thus:  . The Babylonian method differed from the Egyptian in that the multiplicative principle was used in writing 4,000 and 200. If the zero be placed between the two and four, the same symbols are still used; but not so with Ahmes and the Babylonian writers. Following the representation of 4,000, Ahmes would have used two symbols each representing 10 and then three symbols, each equivalent to 1,  . In representing 4023, the Babylonian scribe followed "4,000" with two "tens" and then wrote "three,"  . Similarly, in writing 4004 the Egyptian followed the four lotus flowers by four strokes, , and in writing the same number in ancient Babylon the four wedges would have followed immediately after their 4,000,  . In other words, there was no concept of local or place value. This made it impossible for the early mathematicians to use our modern method of multiplication or of long division, since both presuppose place value. These were both slow processes with the Egyptians and Babylonians.

The world was yet to wait nearly three millennia for a zero and place value. With this "Columbus egg" and the nine Hindu-Arabic number symbols, the development of mathematics was greatly enhanced. Had the early inhabitants of the Tigris-Euphrates and Nile valleys possessed our number symbols, science, as well as mathematics, would no doubt be far in advance of its present development.

Despite the fact that the Egyptians and Babylonians were handicapped by the absence of zero and local value, various tribes and social groups were far inferior to them in the development of number. In fact, languages have existed in which there were no pure number words. The Tacanas of Bolivia have no numerals except those borrowed from other tribes or from the Spanish.[10] The Chiquitos of the same country had no real numbers before they contacted the white man, but expressed their idea for "one" by a word meaning "alone."[11] A few other South American languages are almost as destitute of numeral words. The entire number system of the forest tribes of Brazil appears to be limited to three.[12] The Encabellada of the Rio Napo have only two distinct numbers; meaning "one" and "two"; the number words of the Puris are three; meaning "one," "two," and "many";[13] and the Botocudos count "one" and "many." The Campas[14] of Peru possess separate words for "one," "two," and "three"; but when they count beyond three they proceed by combinations, as one and three for four, one and one and three for five—expressing any number beyond ten as "many." The Conibos[15] had

[10] L. L. Conant, *The Number Concept*, New York: Macmillan and Co., 1923, p. 2.

[11] D. G. Brinton, *American Race*, New York: N. D. C. Hodges, Publishers, 1891, pp. 296, 359.

[12] L. L. Conant, *op. cit.*, p. 7.

[13] Edward B. Taylor, *Primitive Culture*, New York: Brentano's, 1924, I, 243.

[14] L. L. Conant, *op. cit.*, p. 22.

[15] P. Marcoy, *Travels in South America*, London: Blackie, 1874, II, 47.

only two number words when the Spanish invaded South America. In the languages of the extinct or rapidly disappearing tribes of Australia, we find a remarkable absence of numeral expressions. In the Gudang[16] dialect only two numbers are found, meaning "one" and "two." Among the natives of Port Essington[17] and in a number of other instances the numerals stop at three.

Not one of the tribes referred to above developed a civilization. The transaction of business, excepting in an extremely crude way, was impossible. A member of a forest tribe of Brazil could not ask another tribesman to exchange four fish for a fowl unless he used the sign language. A Puri could not demand of a compatriot five of one kind of fruit for three of another. A transfer of goods only on a small scale was possible because of the absence of a number system. Trade among the different tribes was next to impracticable. Hence the transfer of knowledge from one tribe to another could have taken place only by an extremely slow process.

These people represented a point near the beginning of the curve of educational development. Their knowledge of mathematics and other academic fields placed them on a rung near the bottom of the ladder. With reference to the Egyptians and Babylonians, these people represented the antipodes in mathematics and general learning as much as in actual geographic location.

Let us now observe some of the mathematics and general knowledge of a people who had made a number of steps toward the development of a civilization, but still far from an advanced social culture. These people inhabited the vast area north of our southern border long before Columbus discovered America. A study of the habits and customs of the American Indians north of Mexico before the intervention of the white man brings to light many interesting examples of their use of various arithmetical and geometrical concepts and principles, although the absence of an adequate symbolism very greatly limited their mathematical progress.[18]

Counting on the fingers was practiced universally by the Indian tribes of this region. In fact, man's ten fingers have left their permanent imprint everywhere.[19] None of the American Indians north of Mexico had an advanced system of number symbols. Other than the spoken word only a few symbols have been found, and these are on buffalo robes, grave markers, tatooing, and other mnemonic pictographs. The most frequently used method for representing numbers was by the repitition of single strokes, that is, the additive principle in its most elementary form.[20] The Dakotas represented a count of one by a vertical line and a count of one hundred by one hundred such marks.[21] The use of notches cut in wood was frequent in the middle west, in California, and on Puget Sound. At the San Gabriel mission in California every tenth notch extended entirely across the stick. The Creeks used nine vertical marks to indicate nine, but represented ten by a cross. A number of regular systems extended to include one thousand, but in many cases larger numbers were not introduced until after the advent of the white man.

In the Yucatan Peninsula was a tribe of Indians which possessed, long before the birth of Christ, a number system that

[16] E. M. Curr, *Australian Race: Its Origin, Language, and Customs*, London: Trubner; 1888, I, 282.

[17] J. Bonwick, *The Daily Life and Origin of the Tasmanians*, London: Low, 1870, p. 144.

[18] F. L. Wren and Ruby Rossman, "Mathematics Used by American Indians North of Mexico," *School Science and Mathematics*, XXXIII, 1933, 363.

[19] Tobias Dantzig, *Numbers, the Language of Science*, New York: The Macmillan Co., 1933, p. 9.

[20] W. C. Eells, "Number Systems of North American Indians," *American Mathematical Monthly*, XX, 1913, 271–272.

[21] H. R. Schoolcraft, *Historical and Statistical Information Respecting the History, Condition, and Prospects of the Indian Tribes of the United States*, Philadelphia: J. B. Lippincott, 1852, II, 178.

extended beyond one thousand. These people used two different methods of writing numbers. In their codices are found symbols for one to nineteen expressed by bars and dots: each dot represented one unit and each bar represented five. The additive principle was used in forming these symbols, and the principle of local value entered in writing twenty, which was expressed by a dot placed over the symbol for zero. The numbers were written vertically, the lowest order being placed in the lowest position. Accordingly, thirty-six was expressed by the symbol for sixteen (three bars and one dot) in the *kin* (lowest order) place, and one dot representing twenty placed above sixteen. The highest number found in the codices is, in our notation, nearly twelve and a half millions.

The second number system employed the zero, but not the principle of local value. In this system each of the numerals from zero to thirteen is expressed by a distinct type of head. From fourteen through nineteen the head numerals are expressed by the application of the essentials of the head for ten to the head for four to nine inclusive.[22] Both of the Maya number systems were essentially vigesimal. They had a unique word for each positive integral power of twenty up to and including the fourth power.[23]

Tribes employing the decimal system expressed "one hundred" by a unique word or by such forms as "completed," "stock of tens," and "ten times ten"; and "thousand" by "ten times ten times ten," "ten times one hundred," "big hundred," "old man hundred," and "large stock of tens."

The Greenland Eskimo uses "other hand two" for seven, "first foot one" for eleven, "other foot three" for eighteen, and similar combinations up to twenty, which is "man ended." The Unalit employs the same base to twenty; but forty is given by "two sets of animals' paws," sixty "three sets of animals' paws," and so on to four hundred, where there is an interesting change in the formation of this primary base (twenty times twenty) from animals back to man, four hundred being "twenty sets of man's paws."[24] Wintum alternates the decimal and vigesimal bases; twenty, forty, and sixty being "one Indian," "two Indians," "three Indians," while thirty and fifty are "three times ten" and "five times ten" respectively.[25] Kopiagmiut is quinary to ten, decimal for the formation of the odd tens and vigesimal for integral multiples of twenty.[26] The Haida number system is alternately quinary-decimal and quinary-vigesimal to one hundred, then vigesimal, "four hundred" being "twenty times twenty times one" and "eight hundred" being "twenty times twenty times two."[27] The Santa Barbara Indians employed the quarternary system reaching to eight and in forming some numbers above eight, "twelve" being "three times four" and "fifteen" being "twelve plus three."[28] The Cuchan words for three, six, and nine are *hamook*, *humhook*, and *hum-hamook*.[29]

The Coahuiltecan of Texas is a very interesting example of the use of a number of bases. It exhibits binary, ternary, quarternary, quinary, decimal, and vigesimal bases.[30] Represented in Hindu-Arabic some of its numbers are $1,2,3=2+1$, $6=3\times2$, $7=4+3$, $8=4\times2$, $9=4+5$, $11=10+1$, $12=4\times3$, $13=12+1$, $15=5\times3$,

[22] S. G. Morley, "An Introduction to the Study of the Maya Hieroglyphs," Bureau of American Ethnology *Bulletin* 57, Washington: Government Printing Office, 1915, p. 96.

[23] Moritz Cantor, *op. cit.*, p. 9.

[24] E. W. Nelson, "Eskimo About Bering Strait," *Eighteenth Annual Report of the Bureau of Ethnology*, 1896–1897, p. 238.

[25] R. B. Dixon and A. L. Kroeber, "Numeral Systems of Languages of California," *American Anthropologist*, IX, 1896, 675.

[26] E. Petitot, *Vocabulair Française-Esquimau*, Paris: Maisonneuve Freres, 1876.

[27] C. Harrison, "Haida Grammar," *Royal Society of Canada Proceedings and Transactions*, I, 1895, 142–143.

[28] W. C. Eells, *op. cit.*, p. 296.

[29] J. H. Trumbull, "On Numerals in American Indian Languages," *Transactions of the American Philological Association*, 1874, p. 74.

[30] A. Gallatin, *Transactions of the American Ethnological Society*, 1845, I, 1.

$17=15+2$, $30=20+10$, $40=2\times20$, $50=40+10$.[31]

The additive and multiplicative principles existed in the number systems of many North American tribes. Cantor states that addition and multiplication are two methods of counting as old as the formation of number words.[32] The languages of these American tribes verify the statement by Fink that subtraction is very rarely used in the verbal formation of number words.[33]

The number words of the North American Indians were often cumbersome and difficult to use. Examples of an unwieldy system can be seen in the combination *lagwau swaunshung wok swaunshung*, which was used by the Haidas to represent twenty-one, and in *wich a chimen ne nompah sam pah nep a chu wink ah*, which was used by the Sioux to express twenty-nine. The Tshimshians of British Columbia used seven sets of numerals. Each set was used for one of their seven distinct classifications of objects.[34] For example, one set of numbers was used in abstract counting, another set for round objects and division of time, etc. The Takulli dialect of the Athapascan language presents another example of the modification of number words.[35] For example, *tha* was used to represent three things; *that*, three times; *thauh*, in three ways; *thane*, three persons; *thatser*, in three places; and *thailtoh*, all three things.

These Indian tribes north of Mexico give evidence to the fact that some of the elements of mathematics can be developed with a crude number system. Also their arithmetic and geometry substantiate the statement that mathematics can not evolve rapidly where a people depend on a cumbersome and unwieldy system of number words and number symbols. It is interesting to note that these Indians were further advanced in the development of a civilization than were those heretofore mentioned of South America and the Australian aborigines who had only the rudiments of a number system. The use of one hundred strokes to represent one hundred is an advancement far beyond the absence of number symbols. Such representation of one hundred is, however, not only much inferior to the Babylonian and Egyptian symbols for the same number, but is indicative of less cultural development.

The arithmetic of these Indian tribes appears to have been confined to addition, subtraction and in a few cases simple multiplication with integers. Only a relatively small number of tribes had any conception of fractions. Their failure to evolve an adequate system of number words and number symbols prevented their advancing far beyond the simple fundamentals. Their knowledge of geometry was also meager. The geometric forms employed in the building of their houses were principally rectangular and circular. McGee and Thomas believe that the Indians had some method of drawing one line perpendicular to another.[36]

All the essentials of our modern elementary arithmetic, including numeration by position and a symbol representing zero, had been devised by the ancient Mayas two millennia ago, and at least five centuries before the Hindus had evolved the fundamentals of the so-called Arabic notation in India.[37] Little progress has been made in deciphering the Maya

[31] W. C. Eells, *op. cit.*, p. 297.

[32] Moritz Cantor, *op. cit.*, p. 8.

[33] Karl Fink, *A Brief History of Mathematics*, Chicago: The Open Court Publishing Co., 1903, p. 8.

[34] Franz Boaz, "First General Report of the Indians of British Columbia," *Report of the British Association for the Advancement of Science*, 1889, pp. 880–881.

[35] F. W. Hodge, "Handbook of American Indians North of Mexico," Bureau of American Ethnology, *Bulletin* 30, Part I, Washington: Government Printing Office, 1907, p. 354.

[36] W. J. McGee and Cyrus Thomas, *Prehistoric North America, the History of North America*, Philadelphia: George Barrie and Sons, 1905, XIX, 376.

[37] S. G. Morley, "Yucatan, Home of the Gifted Maya," *National Geographic Magazine*, LXX, 1936, 595, 597.

glyphs excepting those pertaining to their calendar and chronology.[38]

Goods were exchanged by members of the same tribe and business and social communion took place between members of different tribes. These North American aborigines transacted business on a much smaller scale than did the citizens of ancient Babylon. To a limited extent, there existed a division of labor which is vitally essential to business intercourse. This division of labor was insufficient to necessitate the exchange of large quantities of goods.

Unlike the forest tribes of Brazil, the progress of learning among the Indians north of Mexico was not prevented by a lack of energy. Their war-like nature precluded peace and harmony, which are always necessary to the production of leisure. Without leisure, the existing knowledge can increase only by a very slow process. Had peace and harmony existed among the early Americans, as it did in ancient Babylon and Egypt, the American aborigines would have had a much different general history, as well as a much different history of mathematics.

The preceding discussion evidences the fact that mathematics had its inception at a very remote date. The study of astronomy was also begun early and in this subject laws of nature were open to discovery by observers, endowed with clear vision and persistence of purpose. This science started with simple and familiar concepts which resulted from inductive thought. Its advancement was stimulated by clarity and definiteness of ideas of space and figure, time and number, and motion and reoccurrence.

Like primitive men everywhere, the Egyptian peasants began at a very remote date to scan the heavens and to observe the stars and the moon. There is probably no other region in which Sirius is such a bright and noticeable spectacle in the evening sky. Aristotle related that the Egyptians attempted scientific observations of the heavens about 2200 B.C. Long before this, however, they observed the periodic changes of the moon, but the lunar month contributed nothing to the determination of the length of the year. The total incommensurability of the stellar or solar year and the so-called lunar year could not be discerned until the approximate length of the solar year had been determined and the fact that there is no such thing as a lunar year recognized. The length of the stellar year as measured by successive sunrise reappearances of Sirius was first roughly estimated by the Egyptians as 360 days more than six millennia ago.[39] As their observations of the heliacal rising of Sirius accumulated, they discovered that the solar year was longer than 360 days. In 4241 B.C. an unknown ruler of prehistoric Egypt added five feast days at the end of the existing 360-day unit, making a convenient calendar of 365 days. This new unit was divided into eleven months of thirty days each and one month of thirty-five days, the months being grouped into three seasons of four months each.

A calendar dominated by the lunar month existed in Babylon as early as the fourth millennium B.C. Instead of five days each year, however, the Babylonians intercalated an entire month every few years. The insertion of an extra month was originally the result of a royal command. Whenever the king noticed that "the year hath a deficiency," he ordered the insertion of an intercalary month.

Babylonian tablets of some five millennia ago contain astronomical reports and astrological deductions based upon them. These tablets contain many lists of stars, and observations of the moon, planets, and comets. The early Babylonians made angular measurements of position, determined the period of revolution of the

[38] Florian Cajori, "The Zero and the Principle of Local Value Used by the Maya of Central America," *Science*, XLIV, 1916, p. 715.

[39] James H. Breasted, "The Beginning of Time-Measurement and the Origin of Our Calendar," *Scientific Monthly*, XLI, 1935, 293.

planets, and predicted successfully eclipses of the sun and moon.

The week of seven days and the day of twenty-four hours had their origin in Babylon. The inhabitants of the Tigris and Euphrates valleys divided the hour into sixty minutes and the minutes into sixty seconds. They divided the circle into three hundred and sixty degrees, the degree into sixty minutes of arc, and the minute into sixty seconds. The savants of today measure time and angles in the same units as did the Babylonians and Sumerians of some five millennia ago. These units enabled the scientists of ancient Babylon to record and accumulate astronomical data.

In contrast to the Egyptians and Babylonians, there are a number of South American and Australian tribes who have not used the periodic changes of the moon in measuring time, neither have they used the heliacal rising of any star in determining the length of the year. In fact, there are several primitive tribes who have no conception of a year, but who recognize half-year periods, corresponding to the wet and dry seasons, or to the monsoon winds, which reverse their directions semiannually.[40] The Indians of the Orinoco anticipate the rains when the monkeys scream at midnight or at the break of day, when the yams which have lost their leaves suddenly grow green again, and when the Pleiades first appear in the eastern sky before sunrise. The Chinhwan of Formosa know that a new year has dawned when a certain flower blooms again. The Tupi of Southern Brazil call the year "cashew-tree," which blooms once a year and bears a fruit used in the making of wine.

The most important bases for calculating time, used by the Indians north of Mexico, were the changes of the moon and the seasons, and the succession of days and nights. Four seasons were generally recognized, although some tribes distinguished more and some less.[41] One tribe counted twelve moons, with some extra days, as a year. Another tribe used thirteen. The Creeks intercalated an extra moon every second year, thus averaging twelve and one half moons in each year. The Sarcees advanced from moon counts by stages of five to six and then to twelve months of thirty days each long before the coming of the white man to the North American continent.[42] Some tribes divided the day into three parts, others used four, and one tribe had a day of nine divisions. The Choctaws expressed a fraction of a day as the time required for the sun to traverse the distance between two parallel lines drawn on the ground.[43] The Indians of South Carolina used the number of handbreadths the sun was above the horizon in determining the time of day.[44]

The methods of reckoning time used by these Indian tribes of the United States and Canada, like their number systems, are indicative of a people low in the evolution of a civilization; much lower than the Babylonians of three millennia before Columbus but far superior to the Australian aborigines of about the same periods.

The Maya made the day, which he called *kin*, the primary unit of his calendar. There were twenty such units following each other in a definite order. When the last day in the list, *Ahau*, had been reached, the count began anew with *Imix*, the first day. This twenty-day sequence continued in endless repetition. Calling the day by one of the twenty names, however, did not sufficiently describe it according to the Maya notation.

[40] Charles F. Talman, "Backgrounds of the Calendar," *Nature Magazine*, XIV, 1929, 22.

[41] H. H. Bancroft, *The Native Races, the Works of H. H. Bancroft*, San Francisco: The History Co., I, 1886, 564.

[42] Moses B. Cotsworth, "The Evolution of Calendars and How to Improve Them," *Pan American Union*, LIV, 1922, 544.

[43] H. B. Cushman, *History of the Choctaw, Chickasaw, and Natchez Indians*, Greenville, Texas: Headlight Printing House, 1889, p. 249.

[44] E. L. Green, *The Indians of South Carolina*, Columbia, South Carolina: The University Press, 1904, p. 45.

In order to complete the name of a day it was necessary to prefix to it a number ranging from one to thirteen, inclusive, as 10 *Ahau* or 3 *Imix*. By this method the Mayan day received its complete designation and found its appropriate place in the calendar. Since there were thirteen numbers and twenty days, any given day could not reappear in the sequence until the two hundred and fifty-nine days immediately following it had elapsed. This method of numbering days was doubtless introduced long after the calendar had assumed a regular form, and probably by the priests, for the purpose of complicating it and rendering it as far as possible unintelligible to the people.[45] This groupof two hundred and sixty differently named days was called *tonalamatl*, or "book of days," by the Aztecs. The Maya name for this unit is unknown. The *haab* or solar year of the Mayas consisted of three hundred and sixty-five days. It was divided into eighteen "months" of twenty days each with a closing period of five unlucky days.[46] The least common multiple of the number of days in the solar year and the *tonalamatl* was another unit of time. This period was called the "Calendar Round" and consisted of fifty-two times 365 days.

Bishop Landa stated that the Mayas knew their solar year to be nearly six hours short of the true length of the year.[47] The Mayan calendar had no elastic period corresponding to our month of February. It is probable that the priests corrected the calendar by adding calculations which showed the exact number of days the recorded year was ahead of the true year at any given time. Thus the Mayan calendar year consisted of three hundred and sixty-five days.

The Mayas in their knowledge of the apparent movements of the heavenly bodies—the sun, moon, Venus, and probably other planets as well—far exceeded both the ancient Egyptian and Babylonian, yet their greatest intellectual achievement was the invention of a chronology exact to a day within a period of more than two millennia, which is as accurate as our own Gregorian calendar.

As we press our way into the meaning of the still undeciphered hieroglyphs, it becomes increasingly apparent that they deal more and more with the subject matter of astronomy.[48] This science, never wholly wanting in the culture of the most lowly aborigines and often occupying a large amount of attention among nations, might easily be employed to furnish an index of the intellectual power of group minds. Thus an intelligence test of tribes, nations, and other units of people might be prepared on the true knowledge possessed by these social groups of the movements of the celestial bodies.[49] On the basis of such a test, the Mayas, Egyptians, and Babylonians, the people who early developed mathematics, would surely be certified as a people blessed with creative minds.

[45] Cyrus Thomas, "A Study of the Manuscript Troano," *Contributions to North American Ethnology*, Washington: Government Printing Office, 1882, p. 7.

[46] John Kinsella, "The Mayan Calendar," THE MATHEMATICS TEACHER, XXVII, 1934, 341.

[47] Diego De Landa, *Relacion De Las Cosas De Yucatan*, Paris: Brasseur de Bourbourg, 1864.

[48] S. G. Morley, "The Foremost Intellectual Achievement of Ancient America," *National Geographic Magazine*, XLI, 1922, 129.

[49] Herbert J. Spinden, "Ancient Mayan Astronomy," *Scientific American*, CXXXVIII, 1928, 9.

Purpose of Education

THE PURPOSE of education is not to settle your minds or fill you with unnecessary information or righteous dogma, or reform you, amuse you, teach you a trade, or give you social prestige. You come to college to learn to think—think straight if possible, but to think, always for yourselves—to learn to read, discuss, and understand—and to do this the old disciplines are needed—grammar, rhetoric, logic, and mathematics—but don't let that scare you—for these are only the arts of reading, writing, and reckoning.—*President Robert Hutchins of the University of Chicago.*

Mathematics in Herbart's Philosophy of Education

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MATHEMATICIANS and mathematical philosophers have often called attention to the fact that mathematical discoveries sometimes seem to anticipate important discoveries in physics. This is usually interpreted as meaning that the universe is "rational at the core" or that it was "created by a mathematician." Such speculations are extremely enticing; but I wish to make an entirely different application of this interesting observation at the present time.

It is probably not generally known that Johann Friedrich Herbart thought of mathematics as performing precisely this same function *at every stage of the learning process*, from the moment a few simple sensory experiences begin to fuse with each other in the consciousness of the young child, until the highest philosophical abstractions become clarified in the mature mind. Furthermore, he reversed this process of reasoning by conceiving mathematics in the rôle of anticipating the applications of abstract knowledge on each level of generality.

I shall attempt to indicate as briefly as possible the direction of Herbart's thinking in connection with this problem; and anyone interested in going further with him can find the substance of his thought in his *A B C of Sense Perception*.[1]

Herbart conceived the learning process as *synthesis:* He regarded it as primarily a matter of putting experiences together into systems of related ideas. In this process, the details (concrete imagery at an early stage) are suppressed, and whatever is common to all the elements of thought which compete for the possession of consciousness at any given time emerges as *form* or as abstract idea. Attentiveness to form is obviously a mark of maturity, from this point of view:[2]

> Color at the expense of form is characteristic of all tasteless ornamentation—the cardinal fault of uneducated sight consists in adherence to color—more exactly speaking, it consists in being immersed in the preeminent color, in losing weaker colors at the instance of the stronger—correctness of sense-perception, which is opposed to this fault, consists in synthetically connecting everything that pertains to the form of a thing—It is attention to form to which our vision requires to be especially educated.

Color, as referred to in this statement, is merely typical of sensory content in general. This is where mathematics comes in to play a rôle in the learning process conceived as *apperception*. It shows why Herbart discusses mathematics at length in a treatise dealing with the psychology of sense perception: Mathematics sharpens apperception and anticipates the next stage of learning by bringing the attention to bear upon the *formed* and *quantitative* aspects of the immediate experience. This obviously does not apply to mathematics as presented in complete detachment from the learner's developing experience. In order to perform this function, it must be very closely connected with the immediate interests:[3]

> . . . In order to furnish preparatory exercise in thinking, mathematical reasoning must not be a peculiar kind of thinking, but it must follow the course naturally and universally taken by sound understanding which is not hindered in its reflections by accidental disturbances

Herbart states further that mathematical reasoning, as concerned with these *nascent forms*, should anticipate apperception just enough to hasten the process, but not enough to give the "presented mate-

[1] A somewhat more practical treatment of mathematics by Herbart can be found in Herbart's *Outlines of Educational Doctrine*, translated by Lange and de Garmo, Ch. iii.

[2] Herbart's *A B C of Sense Perception and Minor Pedagogical Works*. Translated with introductory notes and commentary by William J. Eckoff. New York. D. Appleton and Company, pp. 134–5.

[3] *Ibid.*, p. 160.

rial" an aspect of strangeness or irrelevancy. In this light, the foregoing observation concerning the *inherent rationality* of the universe suggests that the upper level of complexity and abstractness in pure mathematics is often not far in advance of that in mathematical physics.

Herbart's argument does not imply that mathematics is indispensable to the process of learning; but it does mean that it hastens the process when it is used in the right way. It is the hard-edged tool with the aid of which the learner is able to trace out the thought structures which are ready to be separated from the gross materials of experience. It merely *points the way* for the process of making abstractions by calling attention to the *forms* which are next in the line of intellectual discovery. *Apperception*, which is typical of all learning, according to Herbart, moves toward an increased consciousness of *form*. *Synthesis*, as Herbart conceived it, is the process of bringing about a harmony between existent elements of experience. Now, mathematics hastens this harmonizing process either by focusing the attention directly upon the quantitative aspects of experience or by cultivating the tendency to seek out these aspects. From his point of view, it is easy to see how *quantitative forms* emerge as the result of partial suppression of the elements. Elements cannot co-exist in consciousness with all their qualitative details; hence there must be a gentle suppression of *irrelevancies* in order for synthesis to take place. This is the economy of mind from Herbart's point of view.

Now, according to Herbart's psychology, the *application* of abstract ideas is a sort of a reversal of this process of apperception; that is, it is a matter of putting content back into the abstract, empty *forms* of thought from which the content has been removed by the learning process. Mathematics comes in, again, to aid this reverse process by *anticipating application*. This does not mean that mathematical propositions are to be deduced from philosophical abstractions, but that relevant experiences must be shaped to their quantitative outlines before philosophical principles can be effectively applied. A philosopher is likely to become and remain a visionary thinker unless he is able to *form* the qualitative aspects of experience with sufficient astuteness to give the practical bearings of his philosophical ideas some degree of clarity. This brings out very plainly Herbart's conception of the relation of mathematics to philosophy. It also applies to the application of the abstract principles in physics, for example, as well as to those of philosophy. In fact, it applies on all levels of application just as the reverse process applies to all stages of learning.

Mathematics, according to this reasoning, and also as explicitly stated by Herbart, is essential to the beginning, the middle, and the end of education as a process leading toward abstract knowledge; conversely, it applies to all stages of relating an education, so conceived, to life. Philosophy, on the other hand, necessarily comes near the end of education. Herbart calls the latter "the perfector of education";* and, he says, "the office of mathematics is to obviate the dangers of philosophy." But what are the dangers of philosophy? The purpose of philosophy, as he expresses it, is that it " . . . isolates general concepts and removes them for a time out of the sphere of their actual applicability. . . . The concept thus stript gains clearness and definiteness at the simultaneous expense of the disappearance of limits within which, and of the conditions upon which, it has reality."* This loss of magnitude has the advantage, on the upper levels of thought, of clarifying some of the broader relationships which might be hopelessly obscure in a matrix of experience. In this lies both the danger and the value of philosophy. The danger is that the philosopher will mistake these abstractions for the realities which they represent and from which they have been

* *Ibid.*, p. 151.

abstracted; the value lies in the fact that their clarity and relative simplicity sets them in sharp contrast to existing conditions, thus giving rise to idealism and to the instigation of reforms. The young man who has studied philosophy seriously feels the urge to mold the world *nearer to the heart's desire*. This is one of the reasons why the young man should finish off his education with philosophy.

According to Herbart, then, mathematics is a sort of a bridge which can be crossed in two directions. This bears an analogy with Plato's conception of mathematics as the intermediary between *the world of becoming* and the abstract realm of *true being*. However, this apparent agreement between two *great minds* should not be taken too seriously. The basic assumptions upon which Herbart's treatment of mathematics rests are very different from those of Plato. If their respective thought systems seem to meet at their peripheries, it is not because their cores are identical; and it is the philosophical *core*, in each case, which determines the ultimate meaning.

Mathematics Compared with Literature

Mathematics, in its relationship to the apperceptive and synthetic processes of the mind, is supplemented, according to Herbart, by *biography*. Synthesis, in its strictly moral aspect, consists of effecting a union between *insight* and *volition*.[4] Advancement is made in moral development when the *will*, a tendency to act in accordance with a well-established "Circle of thought,"[5] agrees with what the individual has come to recognize as *ideal* or as *right*. But continued advancement requires additional and higher insights which get their patterns from the observation of human conduct and especially from the reading of the right kind of literature. This is the basis of the so-called *culture-epoch* theory which was worked out in detail by Herbart's immediate successors. The different *epochs* supply the successive easy steps in the formation of moral character. For this reason, the *Odyssey* was considered particularly useful pedagogically because it was supposed to idealize, or to draw into a higher unity, the more or less chaotic and competing tendencies which characterize a particular stage of moral development. The pattern for the new insight must *anticipate*, again, processes of synthesis which are about ready to take place anyway. This way, the process of moral synthesis is hastened. Goodness of character, from this point of view, is largely relative to the period of development; and goodness in anything like the absolute sense must be presented to the boy by slow degrees if it is to remain within the reach of his expanding insights.

Therefore, where mathematics anticipates the process of learning from nature and from man as a part of nature, *biography*, in the broad sense, anticipates learning from *humanity* in the humanistic rather than in the naturalistic sense. In fact, these two supplement each other as different aspects of the total process of learning from both nature and humanity. *Knowledge* of nature and *sympathy* with humanity or with society in relation to the *highest being* characterizes what Herbart described as the well-rounded individual possessing the many-sidedness of interest.

The reversal of the process of learning from humanity, corresponding to the application of rational knowledge through the mathematical or *quantitative* formulation of problems, is not quite as obvious, perhaps, as it is in the latter; but it can be found to exist in the form of the sympathetic understanding of human individuals and of humanity on all levels of maturity, because this is certainly the *practical* import of cultivated feelings. Since *sympathy* is conceived by Herbart as that which transports one into the feelings of another, it appears that sympathetic insight can be very greatly facilitated by

[4] *Outlines of Pedagogical Lectures*, Par. 8.

[5] *Ibid.*, Par. 58: "Volition has its roots in thought, etc." or as sometimes translated, "The will takes its roots in the circle of thought."

bringing the attention to bear upon the idealism which corresponds closely to the moral level of those who are to be the objects of sympathetic understanding. For instance, the spectacle of a mature person reading and discussing a piece of literature with a child can be viewed either as the child finding himself in the characters and attitudes portrayed or as the adult *shaping* his own feelings through the literary forms to be conveyed to the child. The human element in story and biography provides these *forms* through which the purified and intensified feelings of the mature, cultivated person can return again into the varied and common human predicaments from which they were abstracted, without losing any of the *intensity* which they gained as free abstractions.

Perhaps this requires some elaboration in terms of Herbart. The *feelings*,[6] in general, according to Herbartian psychology, arise out of the subconscious mind; they are to the subconscious something like what the *will* is to the conscious mind. Therefore, cultivated feelings arise necessarily out of a well-organized, systematized subconscious. But the fact that feelings are related to the subconscious rather than to the conscious mind enables them to acquire a certain degree of detachment from "magnitude" similar to that attained by philosophical *ideas*. There is no reason to believe that these abstract feelings cannot be cultivated by the proper techniques just as philosophical abstractions are cultivated and clarified by the philosophical technique. It might be said, then, that where philosophy clarifies *ideas* and nurtures idealism by projecting the *will*[7] on a high level, "feelings," such as those growing out of the immediate family relationships,[8] can also be intensified by the aesthetic presentations of them as found in the best *biographical* literature.

As a reversal of this process of abstracting and purifying the feelings, it is suggested that the best *containers* through which these feelings can be conveyed to humanity can be found in good literature; and this is similar to the way in which the mathematical or *quantitative* formulation of ideas restores what Herbart calls "Magnitude" to the abstractions of philosophy.

As applied to the sympathetic understanding of the people of any given society, this means that the first essential step is to focus the attention upon the typical *human situations* which are dramatized in their history, biography, mythology, and religion,—not solely, or even mainly, as a scientific study of their customs and institutions,—but as idealized in their real literature. The ground for this in Herbart's psychology is not hard to find; the aesthetic arrangement of experience simply prepares it for easy entrance into the mind which means with the least possible number of conflicts. All assimilation of experience involves *suppression;* a considerable part of the work of suppressing is done in the writing, if the writer is a genius, before the *content* is presented to the mind of the reader.

This at least suggests a relationship between the *aesthetic* and the *psychological*[9] arrangement for both the abstraction and the application of *feelings* which is somewhat similar to that which has been described as existing in Herbart's thought between the *mathematical* and the *psychological* arrangement for the abstraction and application of ideas. It might almost be pressed far enough to imply that when *feelings* are the primary concern, the *aesthetic*[10] approaches or equals the *psychological;* and that where ideas are involved the mathematical approaches or equals the psychological. This, to most people, would seem quite paradoxical, and far from what they would expect to deduce from Herbart, who was the father of the psychological movement.

[6] G. E. Stout, *The Herbartian Psychology* (II). Mind. Vol. 13, pp. 487–91, Oct. 1888.

[7] The *will* as *rooted* in clear and lofty ideas.

[8] *Outlines of Pedagogical Lectures*, Par. 236.

[9] The *psychological* arrangement is for the purpose of making *learning* easy.

[10] "Periods which no master has described, whose spirit no poet breathes, are of little value to education." Herbart's *Science of Education*. Felkin translation, p. 74.

The Effect of Remedial Work in Reading Comprehension upon Algebraic Achievement

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Need for Reading

When the United States declared war against Germany, four million men were chosen for military and naval service. Many thousands were tested in ability to read. Two types of tests were given: the Alpha, which required ability to read and write; and the Beta, for foreigners and illiterates. Since about one-quarter of the men judged were unable to take the Alpha tests, there must have been more than one million of our soliders and sailors who were unable to write simple letters or to read a newspaper with ease.

The army tests furnished impressive evidence in an alarming scale that results of school work in reading should be improved.

In this country, reading has long been recognized as the most important subject that is offered in any curriculum, for in the ability to read lie the paths to all other fields of human knowledge. Every subject in a curriculum should conform to the highest ideals of educational practice. Fundamental, however, to the whole scheme of adjustment is the proper conception of the significance of reading and the important rôle that this subject plays. The progress which pupils make in school depends, to a large extent, upon the success with which they master content subject matter, and this in turn depends upon ability to read independently and intelligently.

With secondary education holding the position that it does in the eyes of the American people, it can not afford to give its pupils less than the proper tools for learning. The tool most frequently applied to secondary school subjects is reading, and a surprisingly large number of those who enter these schools are decidedly deficient in this requisite.

Good reading habits are essential to accurate, creative thought and effective participation in life. The more efficient one becomes in the art of reading, the better is he able to understand and work with his fellowmen. It has been estimated that if every working adult were given just one month's training in how to read, the saving to the nation in increased efficiency would be five billion dollars a year![1]

The numerous studies made during the past few years clearly show that pupils who enter junior high school, senior high school, and even college, vary widely in reading achievement. As a general rule, the scores made on reading tests range from the third-grade norm or below to the twelfth-grade norm. Recent investigations have also disclosed that pupils who rank below the sixth-grade norm in reading are seriously handicapped in studying the reading materials usually prescribed in junior and senior high schools.

It is obvious that schools face serious responsibilities in providing instruction appropriate to the needs of poor readers.

The Problem

The purpose of this study was to discover the effect of reading comprehension, by special training, upon the algebraic achievement of a group of ninth-grade pupils; and to determine, by scientific procedures, the extent to which this increase in achievement is related to the comprehension in reading by these pupils. Scientific studies have been made, and results have shown that reading comprehension can be improved in a surprisingly short time.

[1] Hovious, Carol, *Following Printed Trails*, Chicago and New York, D. C. Heath and Company, 1936, p. 5.

Much current discussion has been related to the importance of high school mathematics. This may be due, in part, to the number of failures in this subject and frequent dislike for the subject. What is the cause of so many failures?

Silent reading for comprehension becomes increasingly important as the pupil leaves the general training of the elementary and secondary schools, and enters the specialized fields of colleges and universities.

It may be that lack of comprehension of words, sentences, and paragraphs—which are troublesome to many pupils—is the stumbling block that makes algebraic achievement difficult, and not so much the "language" of the subject itself.

It was therefore the purpose of this study to determine:

(1) the improvement made by ninth-grade algebra pupils in their general reading ability, due to intensive remedial instruction; and

(2) the improvement which this may effect in the algebraic achievement of these same pupils.

The Procedure

The data upon which this study was based were derived from the results of initial standardized tests and retests, given to the pupils enrolled in the ninth-grade algebra classes of a town of approximately twenty-five hundred population, in September, 1936, and again in May, 1937.

Upon the basis of the initial test scores, of which there was a battery of four tests, the pupils were ranked and divided into two groups as nearly equal as it was possible to make them. The two groups were those who were given remedial instruction in reading comprehension twice weekly during the regular algebra recitation period, and those for whom remedial instruction was not provided. In order to form the two groups for this experiment, the general achievement score, educational quotient, accomplishment ratio, intelligence quotient and score from the algebra prognosis test were each ranked separately for each pupil. The average of these rankings was then made, and upon that basis the two groups were formed. The method of ranking was that of assigning positions to the scores as arranged according to size of score. The largest score was given first position, the next largest the second position, and so on, prior to remedial instruction.

As determined upon the basis of equating, the remedial group was inferior to the non-remedial group in average ranking prior to remedial instruction. The mean accomplishment ratio rank of the remedial group was also inferior to that of the non-remedial group, which means, in brief, that they had not been assimilating knowledge as well as had the non-remedial group. The remedial group included more pupils below an intelligence quotient of 100 than did the non-remedial group.

As a result of remedial instruction in reading comprehension, in the remedial group four pupils were below the ninth-grade level in reading comprehension, as compared with sixteen pupils prior to remedial instruction. The non-remedial group had four pupils below the ninth-grade level, as compared with ten at the beginning of the term.

The rate of reading was changed from twenty-two pupils below the ninth-grade level to ten for the remedial group, and from nineteen pupils to thirteen for the non-remedial group.

The non-remedial group showed some gain, as may be expected with any group; but the remedial group made the greater gain in overcoming deficiencies, because of the greater number in that group who were below the ninth-grade level in reading ability prior to remedial instruction.

There was a mean gain in grade level of 2.7 for the remedial group, and 1.8 for the non-remedial group. These figures represent a gain in percentage of 30 for the remedial group and of 18 for the non-remedial, or a total gain in percentage of 12

in favor of the remedial group in reading comprehension.

The remedial group increased its mean grade level in rate of reading by 1.7 grades and the non-remedial group increased its mean grade level by only one grade.

The town and the country pupils of the two groups were also compared. It was found that the percentage of decrease in deficiencies was, in the remedial group for the town pupils, 25.9; for the country pupils, 18.5. In the non-remedial group the percentage was 11.1 for the town pupils and 10.9 for the country pupils.

Before a comparison could be obtained of the algebraic achievement made by either group, it was necessary to transform the scores made on the standardized tests into standard scores. This procedure was necessary because of the fact that a different number of possible points could be obtained on the two tests that were given, one at the end of the first semester, and the other at the end of the second semester.

Each test had two parts. On Part I of the first test, the remedial group made a sum of 1.80 in standard scores, and the non-remedial group made a sum of 2.78. On Part II, the sum was 1.31 for the remedial group, and .29 for the non-remedial group.

For the second test, the remedial group had a sum in standard scores on Part I of 3.27 and on Part II, 1.42. For the non-remedial group, the sum was 2.72 for Part I and .17 for Part II.

This represents a gain in percentage of 81 on Part I for the remedial group, and a loss for the non-remedial group amounting to two.

On Part II, there was a gain in percentage of 8 for the remedial group, and a loss in percentage of 70 for the non-remedial group.

The chances range from fifty-two to sixty in 100 that the true differences of the obtained scores are greater than zero.

The coefficient of correlation (r) was then computed by using the formula:

$$r = \frac{Sxy}{\sqrt{Sx^2}\ \sqrt{Xy^2}}.$$

where Sxy represents the sum of deviation products,

Sx^2 represents the sum of the squares of one set of deviations, and

Sy^2 represents the sum of the squares of the other set of deviations.

When the formula was solved, it was found that the coefficient of correlation between reading comprehension and algebraic achievement was .70 for the remedial group, and .45 for the non-remedial group.

By reason of the fact that the one group obtained a coefficient of correlation between reading comprehension and algebraic achievement of .70, and that the other group obtained a coefficient of .45, there is evidence of the relationship and effect that general reading comprehension has upon algebraic achievement. The remedial instruction resulted in improved reading, which in turn effected a higher coefficient of correlation with algebraic achievement.

In other words, a higher coefficient of correlation between reading comprehension and algebraic achievement was brought about by means of remedial instruction in the former.

From the above, it is evident that it is high time we are becoming cognizant of the mental scores of our pupils, and remedying those that are deficient. It is a known fact that unrest finds its beginning in lack of understanding. If our citizens are to be directed each according to his aptitude, we must give to them the tool with which they can reach their goals—the simple art of reading. Until that time when they have been given this tool, we shall be surrounded with misfits and inferiority complexes and pseudo-lassitude, which have a telling effect upon society.

Enrichment Materials for First-Year Algebra

By David W. Starr
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General Aims of Mathematical Education

The place of mathematics in the secondary school curriculum has been a subject of very much discussion the last few years. Just what are the aims of a mathematical education? Let us paraphrase such aims as suggested by J. H. Minnick. We should give each individual such a knowledge of the subject as will enable him to understand the exactness and force with which mathematics works and the parts which it plays in solving the problems of nature and which makes it possible for man to turn the elements of nature to his own use; we should develop such fundamental concepts as will enable the student to express his thoughts more clearly and to understand written and spoken language more readily; we should give to each student such a knowledge of mathematics as will enable him to carry on the work of his future occupation as it is now conducted, as will serve as a basis for future preparation if progress in his work should demand it, as will enable him to find new and better ways of doing his work and to recognize a mathematical situation when he sees it; we should require of each student sufficient mathematics to determine whether he will profit by further study of the subject and to select those who will probably be leaders in mathematical thought; and of the select group we should give enough mathematics to keep open the door of specialization in mathematics and in fields dependent upon mathematics.[1]

Specific Objectives of First-Year Algebra

This paper is written with emphasis upon first-year algebra. In the average elementary course for one year what are the specific aims or objectives? These objectives for the student are:

1. To learn the use of the algebraic language; to represent mathematical relations by means of algebraic symbols; to translate words into algebraic representation and vice versa; to express arithmetical rules as algebraic formulae; to understand and use the terms of algebra.

2. To represent statistics by graphs, and to express graphically such formulae as $A = lw$.

3. To use positive and negative numbers.

4. To master the fundamental operations.

5. To master factoring—including trinomial expressions.

6. To understand fractions.

7. To learn to solve simple linear equations.

8. To understand simple radicals and exponents.

9. To understand ratio and proportion.

A Few Difficult Phases of First-Year Algebra

Let us mention a few difficult phases of first-year algebra with which the writer has come in contact and remarks that seem to be a kind of enrichment material.

1. *The idea of functionality.* To institute the idea of functionality—which is so important and which we are prone to barely mention—suppose we have the formula $p = 4s$ for the perimeter of a square. Instead of saying what s is equal to and merely substituting it in the formula—why not make use of some sort of a physical figure on the desk? Short wooden sticks can be used. Then when the sides are increased the fact that the perimeter depends on the length of the sides and how a change in length affects

[1] Minnick, J. H., "The Aims of Mathematical Education," *The Mathematics Teacher*. vol. XIV, pp. 299–303, Oct. 1921

the perimeter can be shown in a more understandable manner.

2. *Equations.* Suppose we have $3x=9$. If we know the value of three x's we must divide to get the value of one x. We have used no word like "axiom." Suppose we have the equation $3x+4x=21$. We must unite. Three x's and four x's give seven x's. This completely does away with the question "What becomes of the other x?" —when we unite $3x$ and $4x$ and obtain $7x$. It fixes attention on the meaning of the symbols.

One of the most harmful words in algebra is "transposition." Suppose we have the equation $4x+9=29$. When the student simply transposes the 9 and changes its sign he is only working mechanically. If we want to know how much $4x$ is equal to we must subtract 9 from it as we have 9 too much—and that means we subtract 9 from 29 to still have an equality. The student will understand the meaning of this procedure and also will understand just what sort of an equation he has worked with means.

When we have $2x=6$, would it not be better to say "therefore" $x=3$ instead of "or" $x=3$? Suppose we have the form of an equation as: $(x-2)(x-3)=0$. Say $x=2$ "or" 3, not "and."

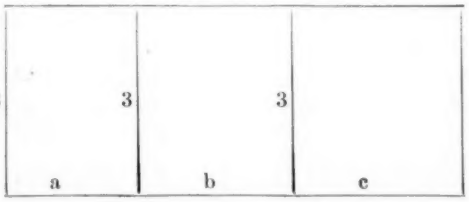
It has come to the writer's attention that some teachers in regard to the clearing of fractional equations use the following form. Suppose we have $\frac{3x}{4}+2=\frac{x}{2}+4$. Then is written this: $\frac{3x+8=2x+16}{4}$. Although it is understood possibly what the 4 means the student is writing an expression that has no meaning whatsoever.

3. *Negative numbers.* Why introduce negative numbers as a very special section? Does this not immediately emphasize the fact that this subject is possibly difficult? At least that is the usual reaction. If negative numbers were presented naturally and right along with positive numbers this over-emphasis is lost. As definite enrichment material a cardboard slide rule made by the student is very useful. On the rule mark off not over twenty equal units on either side of zero. The addition and subtraction of positive numbers are first taken up and then it is a simple matter to naturally take up addition and subtraction of negative numbers. Then addition and subtraction of positive and negative numbers are discussed. The student makes this slide rule before these operations with signed numbers are presented formally.

4. *Objectionable phrases.* Another troublesome phrase is "removing the parentheses." Is this phrase necessary? Suppose we have $3x+(4x-7)$. We must add $4x$ less 7. If we add $4x$ to $3x$ we have added 7 too much so we must subtract 7. We write $3x+4x-7$. Suppose we have $3x-(4x-7)$. We must subtract $4x$ less 7. When we write $3x-4x$ we have subtracted too much as we were to subtract only $4x$ less 7—so we must add 7 and we have $3x-4x+7$.

Another harmful word is "cancel." When we have such an expression as $\frac{3(c-d)}{3(a-b)}$ why not use the phrase "divide through by 3," as that is exactly what we do? We will eliminate somewhat the usual mistake of "canceling" in such a manner as $\frac{3a-b}{3}$.

5. *Factoring.* It has been found useful to use a rectangle divided into—say three parts—and find the area thus: The student knows how to find the area of each small rectangle. The areas of each rectangle are found to be $3a$, $3b$, and $3c$. The area of the whole rectangle would then be $3a+3b+3c$. Let us look at the rectangle as a whole. The length is shown to be $a+b+c$ and the width is 3. Then the area



is $3(a+b+c)$. Thus the simplest case of factoring has been shown the student without a formal rule.

Suppose we have $(x-2)(x-5)=0$. It is a mistake to say we "set" $x-2=0$ and leave it there. Explain to the student "why" we say $x-2=0$ and $x-5=0$. If the product of factors is zero one of the factors must be zero.

6. *Verbal problems.* Verbal or stated problems used should be those that are meaningful and lifelike to the student. Such a problem as the following would occur only very rarely and simply bewilders the student. The problem:

In ten years John will be half as old as his father. In twenty years he will be three-fifths as old as his father. How old is John now?[2]

It is important to stress what is given in a verbal problem. W. G. Brink gives directions to the student as follows:

Directions: Read each problem carefully. Then read each statement (a–e) below the problem and write in the box every letter which stands for a fact given in the problem. Several of the statements may be true. Do not miss any. Refer to the problem as often as you need to.

1. A lady took a train to the city, a distance of 19 miles. There she took another train in which she rode 172 miles. How far did she ride on the two trains?

	(a) Speed of the train to the city
	(b) Distance the lady rode on the first train
	(c) Time she waited for the train to come
	(d) Length of time lady was on train
	(e) Distance the lady rode on second train

2. A factory employing 285 workmen found it necessary to discharge 90 of them. How many workmen were retained?

	(a) The amount paid workmen
	(b) The product manufactured
	(c) Number employed before discharging 90 men
	(d) Number employed after discharging 90 men
	(e) Number of men discharged[3]

The ability to select data is important. Mr. Brink calls attention to four types of verbal problems to be shown the student:

(1) Problems which contain superfluous numbers
(2) Problems which contain superfluous information
(3) Problems which contain insufficient data
(4) Problems which contain superfluous data but lack essential data

The following problems will illustrate the first two types:

1. *A boy made 50 toy boats. He sold 24 of them at 30 cents each, 10 at 25 cents each, 6 at 20 cents each, and the rest at 15 cents each. How much did he receive for those he sold at 30 cents each?*
2. *Mrs. Smith bought a vacuum cleaner for $70 to be paid in monthly installments. Two months she failed to make the payments but each time she made it up with the next payment. How many months was it before the cleaner was paid for if her payments were $5 per month?*[4]

Instead of saying find "the" equation would it not be better to explain to the student that there is not necessarily just one equation to be found but that there are many equations that will solve the verbal problem?

A, *B*, and *C* are famous characters in verbal problems. The story of *A*, *B*, and *C* by Stephen Leacock is very interesting for the student to read or have read to him.

For the most part in the examples used herein x has been used as a symbol. Yet is this particular symbol always necessary? If the student finds it easier to work with other letters why not permit him to do so? Such symbols as p for price, r for rate, l for length, and w for width are found easier to use sometimes than the symbol x.

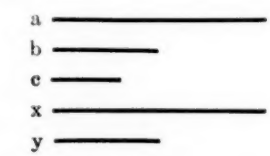
7. *Ratio and proportion.* It seems that ratio and proportion is neglected as a subject—at least little time is spent on it. More time than the usual one or two days (this is about all the time given to it by the usual textbook) should be spent on this subject. There are three ideas in regard to ratio.

First, it is a number as the ratio of the circumference of a circle to its diameter is 3.14 approximately; secondly, ratio tells how many

[2] Brink, W. G., *Directing Study Activities in Secondary Schools*, p. 528.
[3] *Ibid.*, p. 533.
[4] *Ibid.*, p. 535.

times: Two numbers are so related that the first is three times the second, then they are said to be in the ratio of 3 to 1; and thirdly, to find ratio one divides as: "Mr. White earned $27 in 6 days. How much will he earn in 14 days at the same rate?" The pupil ought to be taught to form a ratio expressing how many times as much. The ratio is either 6/14 or 14/6. If the answer is to be less than $27 the result will be 6/14 of $27. If the answer is to be more than $27 the result will be 14/6 of $27. The pupil's judgment will tell him which ratio to use.[5]

8. *Graphs.* Bar graphs may be used in relation to ratio. Suppose we have such a graph as:



line a extending six units
line b extending three units
line c extending two units
line x extending six units
line y extending three units

What is the ratio of the length of a to length of b? How do you find that ratio? The ratio of the line a to line b is $6 \div 3$ or simply 2. This means that a is twice as long as b. We have found the ratio by dividing. The ratio of line b to line a is $3 \div 6$ or $\frac{1}{2}$. This means that line b is $\frac{1}{2}$ as long as a. Is the ratio of line a to line b the same as the ratio of line x to line y?[6]

It seems to the writer that per cent might well be discussed in its relation to ratio and proportion. Suppose the question was to find what per cent of 20 games played by an athletic team is the 14 games it won. Expressed as a proportion we have $\frac{14}{20} = \frac{p}{100}$. The per cent is 70.

To develop a concept of proportion have the student take a large comic picture or photograph and across it draw small squares. Then on his paper he draws larger squares. By locating points in square corresponding to square on picture the main outline is obtained in correct proportion.[7]

At this point it should be mentioned that many teachers neglect the teaching of graphs.[8] Some leave the subject out entirely. Graphs afford one of the most interesting means of showing the student the relation of mathematics to everyday life. The student should collect printed graphs from newspapers and magazines on some topic in which the student is especially interested. Then the student should make graphs on some topic for which the student must collect and arrange his own data such as the student's success on written work in a class or the success of the high-school athletic team.

9. *Fractions.* If the history of fractions and how fractions developed are told the student, he will see much better the need for and the use of fractions. Three interesting and brief booklets under the title: *Achievements of Civilization: The Story of Weights and Measures; The Story of Our Calendar; The Story of Numbers* can be obtained from the American Council of Education, Washington, D. C.

The student has a desire to feel that his work is important and that it is a part of the plan of life. He is pleased to know that he is working with something vital in the makings of civilization. We must not let his interest lag. The teacher has not yet used all the available resources.

The mathematics classroom should be a place that does not have the appearance of being cold and drab. Something should be in it besides the seats, desk, and blackboard. And there are many things that can be put in the classroom to make it an attractive place for work and study. A bulletin board is a very needful article. On this can be posted items from maga-

[5] Allison, John R., "Using Ratio in Problem Solving," *The Mathematics Teacher*, vol. XXIII, pp. 187–88, March, 1930.

[6] Schorling, Raleigh and Clark, John R., *Mathematics in Life*, pp. 229–30.

[7] Engle, T. L., "Developing a Concept of Proportion Before Presenting the Formal Work," *School Science and Mathematics*, vol. XXXII, pp. 268–69, March, 1932.

[8] *Graphs: How to Make and Use Them* by Aiken and Colton, Harpers, is useful.

zines and newspapers that pertain to mathematics.

Pictures, charts, and graphs showing applications of mathematics should be shown in the room. The "Tree of Knowledge," a poster, should be in every mathematics room. One can be obtained from the Business Manager of the Museum of Science and Industry, Jackson Park, Chicago, Illinois, in color, 125″ by 30″, for twenty-five cents postpaid. The six oil paintings depicting the History of Standard Units of Measure are very interesting. These can be obtained from H. G. Ayre, State Teachers College, Macomb, Illinois. Algebraic charts by Edgar Dehn, particularly number one on the quadratic equation, can be obtained from the Columbia Press, 2960 Broadway, New York City.

It seems advantageous to have a few pictures—good ones—of some of the famous mathematicians. Portraits of eminent mathematicans can be obtained from *Scripta Mathematica*, 610 West 139th Street, New York City. Many of these men have interesting lives. E. T. Bell's *Men of Mathematics*, Simon and Schuster, New York, publishers, is very good indeed not only for the student to read, but for the teacher to read from or relate portions from. The students may make reports regarding some of these mathematicians.

The motion picture offers a new field of enrichment material. "Optical Illusions" can be obtained from Visual Sciences, Suffren, New York. The film, "Parabola," is obtainable from Rutherford Boyd, 112 Prospect Street, Leonia, New Jersey.

There should be a good mathematics library containing books on the history of mathematics, reference books, magazines, and books on mathematical recreations and plays. Mathematics clubs can be formed and plays can be put on by its members. Many plays are to be found in the periodical, *The Mathematics Teacher*.

The periodicals, *The Mathematics Teacher* and *School Science and Mathematics*, should be in every professional mathematics library. The *Yearbooks of the National Council of Teachers of Mathematics* are useful.

The teacher of mathematics must be continually improving his background. He should have at his command, not only for his own use, but for his students, a good library that is to be put into use and kept in use. With the enrichment material that is available the real teacher of mathematics can go far in showing the student that mathematics is truly an interesting, useful, and living subject. Let us remember that all these materials are not useful and interesting to every student. Let us, then, make judicious use of these enrichment materials.

In a course with Dr. E. D. Pepper of the University of Illinois much attention was given to available enrichment material. Let us conclude, then, with a somewhat lengthy bibliography that contains valuable resources not only for the teacher of first-year algebra but also for the teacher of other phases of mathematics.

AVAILABLE ENRICHMENT MATERIALS

(Given in aforementioned University course by Dr. Pepper with additions)

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The Story of Time Telling. $.20.
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—— *The Beautiful Necessity*, Rochester, N.Y.: Manas Press, 1922. $2.50.

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THE MATHEMATICS TEACHER

525 W. 120th Street, New York, N.Y.

Criteria for the Evaluation of a Course of Study in High School Mathematics

By SHIRLEY E. BOSELLY, JR., *Lincoln High School, Seattle, Wash.*
and HARRIET HOLBROOK, *Girls' Six Year High School, Portland, Oregon*

A. PHILOSOPHY

1. Does the course present a consistent and practicable philosophy of education?
2. Does it list any guiding principles to be applied in the appraisal of the newer educational trends?
3. Does it differentiate between general education and vocational education?
4. Does it differentiate between social education and individual education?
5. Does it attempt to maintain standards even though secondary education is to be adapted to the individual pupil?
6. Does it call for adequate preparation of secondary teachers in the course?
7. Does it define the duties of the secondary school in coping with the problem of mass education?
8. Is the philosophy functional, progressive and flexible?

B. OBJECTIVES

1. Are the educational aims and objectives stated in non-technical language?
   Are they clearly stated?
   Can they be easily referred to?
2. Are they stated in detail with reference to:
   a. Habits?
   b. Attitudes?
   c. Knowledge?
   d. Pupil's goals, not teacher's?
3. Are the statements of objectives extensive enough to realize pupil needs, both in subject matter and at any grade level?
   Are they grouped into teacher and pupil objectives?
4. What is the basis for the selection of the objectives:
   a. Opinion of the committee?
   b. Report of national committees?
   c. Opinions of educational experts?
   d. Other courses of study, textbooks, subjects?
   e. Scientific investigations?
   f. Experimentation?
   g. Opinion of laymen?
   h. "Frontier Thinkers"?
   i. Newspapers, magazine, statistical reports?
5. Reasons for the selection of objectives:
   a. Frequent occurrence in child life?
   b. Frequent occurrence in adult life?
   c. Importance, even though infrequent?
   d. Interest to pupil?
   e. Can be attained only in school?
   f. Can be measured?
   g. Can be best secured in this course?
   h. Are they selected in accordance with the cardinal principles of secondary education?
   i. Are they selected according to the educational philosophy selected?
6. Does it recognize the basic mathematical objectives previously stated?
   a. Mathematics as a mode of thinking:
      1. Collect and understand facts?
      2. Evaluate facts?
      3. Select right facts?
      4. Relate facts?
      5. Apply facts to daily problems?
   b. Transfer of training may be secured, if definitely taught?
   c. Recreational values of mathematics?
   d. Knowledge as an ultimate goal?
   e. Preparation for future courses?
   f. Preparation for vocations?
   g. Interrelations with other subjects?
   h. Character training such as:
      1. habits?
      2. attitudes?
      3. appreciations?
   i. Increased computational skill?
7. To what degree have the objectives been carried out consistently in the selection, organization, etc., of subject matter and methods?

C. CONTENT

Criteria for the selection of subject matter

1. Does it contribute to the needs of the pupil:
   a. Social?
   b. Home?
   c. Community?
   d. Individual?
   e. School?
2. Does the material appeal to pupil interests as to:
   a. Motivation?
   b. Intrinsic value?
   c. Evident worth?
3. Is the material either:
   a. Concrete?
   b. Vital?
   c. Diagnostic?
   d. Remedial?
   e. Drill?
   f. Exploratory?
   g. Recreational?
   h. Cultural?
4. Is the material of such a type and presented in such a way as to develop profitable and desirable:
   a. Attitudes?
   b. Appreciations?
   c. Habits?
   d. Skills?
   e. Knowledge?

Criteria for Unit Organization

1. Are the objectives so clearly stated that both pupil and teacher can understand them and work together for their realization?
2. Does the unit have coherence between pupil experience and future outcomes?
3. Does it provide for participation of the pupil?
4. Is it practicable in the particular community and school?
5. Does the unit challenge the pupil at the particular grade level, and is it within the pupil's ability?
6. Does the unit reproduce real life situations?
7. Is the unit interesting to the pupil?
8. Are all phases of the teaching unit clearly and definitely organized?
9. Does the unit begin with life situations and proceed to generalizations by the pupil?
10. Does it designate all materials and devices of value?
11. Does the unit provide for individual differences?

Content of the Course of Study

1. Is the subject matter well organized?
2. Does it correlate with other subject matter?
3. Is there a wide range of subject matter?
4. Does it suggest other possible topics?
5. Does it achieve the stated aims?
6. Is the subject matter vital in terms of human experience?
7. Is the material presented functional in terms of human experience?
8. Is it the best material to achieve the educational aims?
9. Is there poper emphasis on:
   a. Drill material?
   b. Problem material?
   c. Knowledge and application?
   d. Modes of thinking?
   e. Attitudes, appreciations?
   f. Skills?
10. Is the material presented according to:
    a. Pedagogical, rather than traditional form?
    b. Difficulty, rather than logical form?
    c. Units or project-problems, rather than chapter form?
11. Is the material a product of:
    a. Fusion?
    b. Integration?
    c. Correlation?
    d. Single subject?

Problems and Activities

1. Are the various problems:
   a. Practical?
   b. Hypothetical?
   c. Cultural?
   d. Based on mental discipline theory?
   e. Graded according to difficulty?
   f. Applicable?
   g. Extensive?
   h. Purposeful?
   i. Interesting?
   j. Self-motivating?
   k. Original?
   l. Satisfying?
   m. Adaptable to individual needs and differences?
   n. Reasonable?
   o. Clear and concise?

Additional Problems and Activities

1. Does the course suggest additional problems and activities which might stimulate valuable cultural and recreational studies:

a. In the school?
b. Outside the school?

2. Are these materials and studies:
   a. Easily obtained?
   b. Timely?
   c. Sufficient in number and kind?
   d. Applicable to local situations?
   e. Capable of correlation within a subject?

Scope of the Course of Study

1. Does the course cover sufficient material for:
   a. College entrance?
   b. Future related courses?
   c. Correlation with other subjects?
   d. Future adult needs?
   e. Pupil satisfaction?

D. Methods

Organization

1. Is the course organized according to a minimum school year?
2. Is there a time allotment within the course for the materials?
3. Is there a maximum and minimum time allotment?
4. Does it state the time of the year for the various materials?
5. Does it prescribe the material according to grade placement?

Problems and Activities

1. Does the course suggest comprehensive activities and materials of instruction so the teacher may achieve the aims listed?
2. Does it suggest sufficient and necessary illustrative material?
3. Are they easily obtainable?
4. Does it help the teacher in presenting and adapting them?
5. Does it evaluate them according to immediate needs?
6. Does it list activities not confined to the classroom?
7. May these be done without teacher participation?

The Text

1. Does the course suggest proper functions of the text?
2. Does it list more than one text?
3. Are library references listed?
4. Can the pupil succeed with a single text?
5. Does the course require drill material?
6. Is it so arranged that the pupil or the teachers may provide their own drill material?
7. Is the drill only incidental to the aims?
8. Is the drill applicable?
9. Does it provide surveys for purposes of diagnosis and orientation?

Teaching Methods

1. Does the course suggest a variety of procedures for attacking the separate situations?
2. Does it suggest the best methods?
3. Does it list the devices applicable?
4. Does it suggest procedures to realize the specific objectives of the course?
5. Does it aid the teacher to plan the work beforehand?
6. Does it give type lessons or units?
7. Does it give relative emphasis on oral and/or written work?
8. Is it conducive to disciplinary control within the class room?
9. Does the method suggest when it may be:
   a. Inductive?
   b. Deductive?
   c. Synthetic?
   d. Analytic?
10. Does the course list the desirable outcome for both teacher and pupil?
11. Are they clearly stated?
12. Can they be achieved by every pupil?
13. Are provisions made for individual differences?

E. Testing Results of the Course

Types of Tests

1. Does the course include standard tests; remedial, diagnostic, mastery and appreciative?
2. Are there provisions for subjective tests?
3. Are there provisions for original tests?
4. Are there references to other standard tests?

Evaluation of Tests

1. Does it help the teacher to select and use these tests to determine outcomes:
   a. As to time allotment?
   b. As to the time to be given?
   c. Mimeographing?
   d. Amount of material to be tested?
2. Is the pupil able to evaluate his own work?
3. Does the test help the teacher to evaluate his own work?
4. Are the tests based on the original objectives of the course?

5. Are the tests representative of the subject matter to be tested?
6. Are the problems clearly stated?
7. Are they arranged in order of difficulty?

F. Bibliography Content

1. Are there sufficient and necessary lists of references?
2. Are they up to date?
3. Are they useful for individual differences?
4. Are there periodical and pamphlet references?
5. Are there suggestions for their use?
6. Are there suggestions for their acquisition?
7. Are there references for club work?
8. Are there references for additional work?
9. Are there references for devices, materials, etc.?
10. Are the references well grouped and classified?
11. Are they listed as to teacher or pupil references?
12. Are they alphabetically arranged?
13. Are they annotated?
14. Are they attractive?
15. Are they easily referred to?
16. Can they be easily revised?
17. Are they listed according to basic or supplementary references?

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General Rating of the Course	Exc.	Above Average	Average	Below Average	Poor	
1. Philosophy						
2. Objectives						
3. Content						
4. Methods						
5. Testing						
6. Bibliography						
7. Mechanical Makeup						
8. Average						

The Cleveland Meeting

Don't forget the Cleveland meeting of the National Council of Teachers of Mathematics on February 24 and 25 at the Carter Hotel. The complete program for this meeting appears on pages 30 through 32 of the January issue of The Mathematics Teacher.

Air Navigation and Secondary School Mathematics

By JOHN KINSELLA, *Newburgh Free Academy, Newburgh, N. Y.*
and A. DAY BRADLEY, *Hunter College of the City of New York*

THE World's Fair of 1939 has the distinction of being the first of its kind to be heralded around the world by men from the air. The flight of Howard Hughes and his men will long be remembered by millions of Americans who followed his progress hour by hour around the globe. The speed of the trip startled the whole nation. The constant radio contact between the plane and the rest of the world was especially remarkable. The flight followed a prearranged schedule with such precision that the distance traveled was only twenty miles greater than the planned mileage.

It is not surprising to learn that mathematics plays a part in accuracy such as this. Not long ago the flights of the China Clipper were given much publicity. In developing a routine of navigation to be followed on their trans-Pacific flights, Pan American Airways made many changes in the methods of classical navigation in order to reduce the time required for the necessary computations. While the determination of a plane's position by means of radio is of great importance, the older methods of *celestial navigation* and *dead reckoning*, so well known to men of the sea, are still fundamental in air navigation. Celestial navigation is the determination of latitude and longitude by observations of the heavenly bodies. Dead reckoning is the determination of the plane or ship's new position by trigonometry when the direction and distance from a previously given position are known. Both dead reckoning and celestial navigation require the use of tables and graphs constructed from the formulas of trigonometry. All of these devices would be insufficient unless the weather were carefully studied before "taking off."

Allowance must always be made for the the effect of the force and direction of the wind on the direction and speed of the plane. The rapid speed of modern planes in contrast with that of ocean going vessels has necessitated many changes in the observations and computations of navigation. Consider the change of position of a plane going 200 miles per hour for a period of only three or four minutes in comparison with that of a steamer making twenty-five knots. While the methods of celestial navigation are based upon practical astronomy and spherical trigonometry, the determination of the effect of the wind on the plane's speed and direction involves only scale drawing, plane trigonometry and graphs.

Two problems of extreme importance in air navigation are: (1) to determine the *wind correction angle*, that is, the angle which the plane must be steered into the wind in order to fly in a desired direction, (2) to determine the *ground speed*, that is, the actual speed made good over the ground or ocean in contrast to the *air speed* which is registered by the speed indicator. In practice the answers to these two problems are obtained quickly from a graph. The data for the construction of this graph can be found by the law of sines of plane trigonometry or by the scale drawing of intuitive geometry depending upon the learner's background. It will be interesting to note in the solution of the following specific problem the importance of exact definitions, which are stressed so strongly in plane geometry teaching.

The writers want to acknowledge their rather free use of *Practical Air Navigation and the Use of Aeronautical Charts* (Special Publication No. 197, U. S. Coast and Geodetic Survey) by Thoburn C. Lyon.

Problem A: To Find Wind Correction Angle and Ground Speed

The cruising speed of a plane is 120 miles per hour and the true course is 75°. The wind is 30 miles per hour from 220°. Find the wind correction angle and the ground speed.

Before solving this problem it is necessary to explain a few technical terms; these terms are illustrated from the data of the problem itself. In measuring the course of direction of flight 90° corresponds to east, 180° to south, 270° to west, and 0° or 360° to north. The meaning of the expression "wind from 220°" is illustrated by Figure 1. The true course is

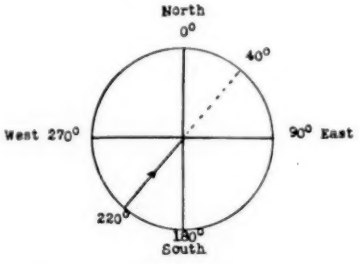

Figure 1

FIG. 1

the direction of the destination from the point of departure. Another important term is the "angle between the wind and true course"; shown in Figure 2. Note

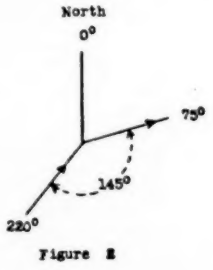

Figure 2

FIG. 2

that the wind vector is directed toward the vertex of the angle, while the true course vector is directed away from the vertex. The angle between the wind and true course is the smaller of the two angles made by these vectors, hence, in Figure 2, it is $220° - 75° = 145°$ (rather than 215°).

SOLUTION I: SCALE DRAWING

The illustrative problem stated above may be solved by scale drawing as is shown in Figure 3. To construct this figure:

1. Let NA be the north-south line through the point of departure, A.
2. Construct angle $NAB = 75°$, the true course.
3. Construct EA from the direction 220° to represent the wind direction.
4. Extend AE to E', making $AE' = 30$ miles, to some convenient scale.
5. With E' as center and a radius of 120 miles, to scale, describe an arc intersecting AB at C.
6. Determine D, the fourth vertex of the parallelogram $AE'CD$.

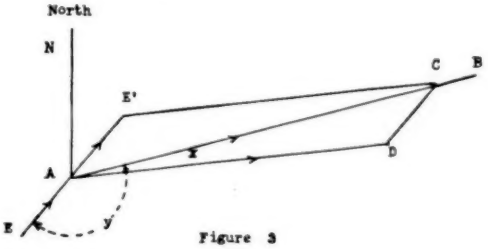

Figure 3

FIG. 3

In order to stay on the desired course represented by AC, the plane must be steered into the wind, in the direction AD. In this case the pilot must steer the plane to the right of the desired course in order to offset the force of the wind driving it off the course to his left; that is, the pilot must steer in the direction AD in order to fly along AC. Angle CAD is called the wind correction angle. As mentioned above, the actual speed of the plane over the ground or ocean, called ground speed, is different from the air speed which is shown on the speed indicator. Ground speed is the resultant of air speed and wind velocity. In Figure 3, the ground speed is represented by the length of AC. For this problem the approximate answers obtained from the figure are:

wind correction angle = angle $CAD = 8°$,
ground speed = length of AC to scale
= 143 miles per hour.

SOLUTION II: BY PLANE TRIGONOMETRY

This problem may be solved by applying the law of sines of plane trigonometry to triangle ADC.

(a) To find the wind correction angle:

$$\frac{\sin CAD}{\sin ACD}=\frac{CD}{AD}, \text{ or}$$

$$\sin CAD=\frac{CD \sin ACD}{AD}$$

$$ACD=180°-EAC=35°.$$

$$\sin CAD=\frac{30 \sin 35°}{120},$$

$$CAD=8°15' \text{ or } 8°.$$

In general, denoting the wind correction angle by x and the angle between the wind and the true course by y:

$$\frac{\sin x}{\sin y}=\frac{\text{wind velocity}}{\text{air speed}}.$$

(b) To find the ground speed:

$$\frac{AC}{AD}=\frac{\sin CDA}{\sin ACD}, \quad \text{or}$$

$$AC=\frac{AD \sin CDA}{\sin ACD}.$$

$$CDA=180°-(35°+8°)=137°.$$

$$AC=\frac{120 \sin 137°}{\sin 35°},$$

$$AC=143 \text{ miles per hour.}$$

In general:

$$\frac{\text{ground speed}}{\text{air speed}}=\frac{\sin (y-x)}{\sin y}.$$

SOLUTION III: BY GRAPH

Using the formula ($\sin x/\sin y=$ (wind velocity/air speed), a graph can be constructed giving values of the wind correction angle at a glance. Figure 4 has been reproduced from *Practical Air Navigation* by permission of the Director of the United States Coast and Geodetic Survey. This graph consists of several curves for various wind correction angles. For greater simplicity the air speed may be taken as 100 miles per hour. To obtain a particular curve a constant value is assigned to the wind correction angle, and the wind velocity corresponding to a given value of the angle between the wind and the true course is computed. For example to plot the curve for a wind correction angle of 10°, the formula is wind velocity $=(100 \sin 10°/\sin y)$. The table below is obtained by giving y the values 30°, 45°, 60°, etc.

y	wind velocity
30° or 150°	34.7
45° or 135°	24.5
60° or 120°	20.0
75° or 105°	18.0
90°	17.4

For given air speed others than 100 miles per hour, the wind velocity must be considered as a per cent of air speed. In solving the problem given in this paper by the graph of Figure 4, the wind velocity of 30 miles per hour becomes 25 per cent of the air speed (120 miles per hour). The angle between the wind and the true course is 145°. Hence it is necessary only to find the intersection of the line representing a wind

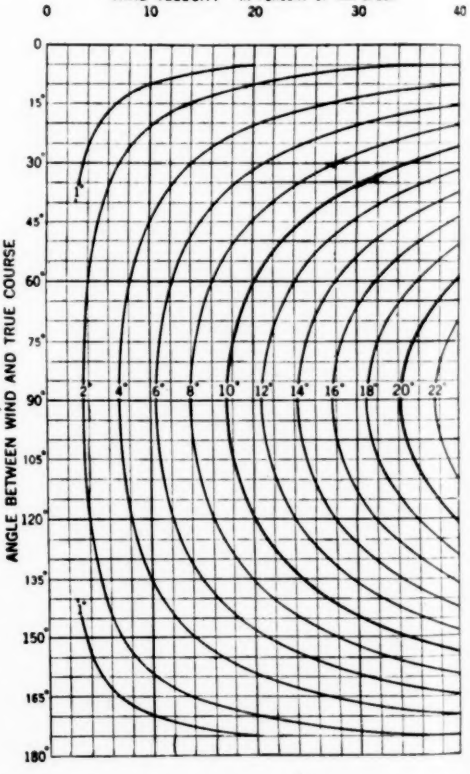

FIG. 4. From *Practical Air Navigation* by Thoburn C. Lyon, U. S. Coast and Geodetic Survey.

velocity of 25 with the line representing an angle of 145°; this intersection will lie on the curve for a wind correction angle of 8°.

The reader may note the speed of the graphic method as opposed to the comparative slowness of the scale drawing or the solution by trigonometry. For finding the ground speed rapidly other curves are calculated for a separate chart according to the second formula given above, ground speed/air speed = $\sin(y-x)/\sin y$. Sudden changes of wind direction and velocity make it necessary that the wind correction angle and the ground speed be re-determined frequently in order to keep the plane on its course. Graphs similar to Figure 4 are used when the plane is in flight.

Problem B: Interception

The problem of finding the course which a plane shall take in order to overtake a second plane, known to be following a given course at a given speed, is called *interception*. This problem occurs when a plane is to be refueled in flight, when planes from different points are to join each other on patrol, and is of importance in military operations. The problem of interception may be solved easily by a figure drawn to scale; the principle involved is the old rule of navigation, that if the direction of one ship from another is constant, the ships will collide.

Suppose in Figure 5 that one plane is following the course AC, and a second

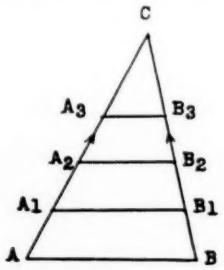

Fig. 5

plane the course BC; A_1, A_2 and A_3, etc. are successive positions of the first plane, and B_1, B_2 and B_3, etc. are the corresponding positions of the second plane. When the first plane is at A_1, the bearing of the second plane from the first will be angle CA_1B_1. If the angles CA_1B_1, CA_2B_2, etc. are equal the planes will always be on a line parallel to AB and hence must reach C at the same time. The method of solving problems of interception graphically is indicated by the following examples.

I. Interception Without Wind

A plane capable of making 140 miles per hour is to leave point A on a course of 250°. At the same time a second plane having a speed of 110 miles per hour is to leave point B. B is 40 miles west and 70 miles south from A. Find the course of the plane from B and determine where the interception will occur.

To construct Figure 6.

1. Draw right triangle ABC with $BC = 40$ miles and $CA = 70$ miles, to some convenient scale.
2. Construct AD in a direction 250° and with a length of 140 miles to scale.
3. Through D draw the line DE parallel to AB.
4. From B with a radius of 110 miles to scale, describe an arc intersecting DE at E.

AD in Figure 6 represents the course and speed of the plane from A. The length of

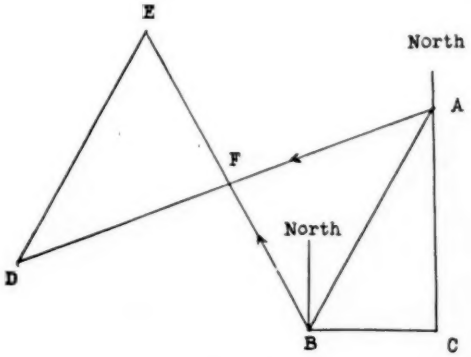

Fig. 6

BE represents the speed of the second plane, 110 miles per hour, and the direc-

tion of BE, the course of the second plane, 315°. Interception will occur at point F. The distances traveled by the planes are AF and BF. Hence,

the time for the first plane to reach $F = AF/140 = 66/140 = .471$ hours,

the time for the second plane to reach $F = BF/110 = 52/110 = .472$ hours.

The times found by this method will generally not agree exactly in the third significant figure.

II. Interception With Wind

Figure 7 illustrates a similar problem of interception where there is a thirty-mile

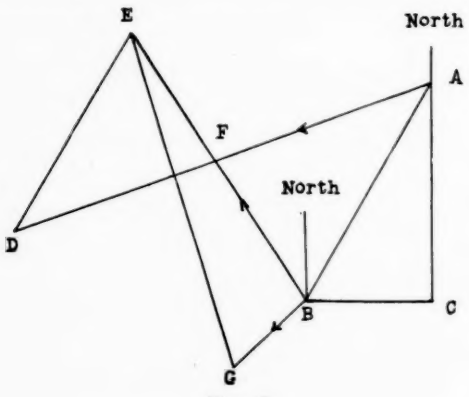

Fig. 7

wind from a direction of 45°. A and B are located as above, the *true course* of the plane from A is 250°, and its *ground speed* is 140 miles. The air speed of the plane from B is 110 miles per hour.

To construct Figure 7:

1. Triangle ABC, lines AD and DE are constructed as in Figure 6.
2. From B draw BG to represent the wind of 30 miles per hour from 45°.
3. With center G take a radius of 110 miles and draw GE intersecting DE at E.
4. Draw BE.

The direction of BE represents the true course of the plane from B, 327°, and the length of BE is its ground speed, 99 miles per hour. The interception will occur at F and, as above, the problem may be checked by finding the times for the planes to reach F.

$$\frac{AF}{140} = \frac{73}{140} = .523 \text{ hours.}$$

$$\frac{BF}{99} = \frac{52}{99} = .524 \text{ hours.}$$

Problems such as the above show that air navigation offers teachers of secondary mathematics an opportunity of vitalizing their teaching. There is no topic more interesting to boys of high school age than that of airplanes. Further applications of mathematics to air navigation will be found in the following bibliography.

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Lyon, T. C., *Practical Air Navigation and the Use of Aeronautical Charts*, Special Publication No. 197, U. S. Coast and Geodetic Survey.

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Back Numbers Wanted!

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6. Vol. III, No. 3, March 1911.

Mathematics in Mechanical Engineering*

By GEORGE B. KARELITZ
Columbia University, New York City

THE USE of Mathematics in Mechanical Engineering is essentially the same as in all other kinds of engineering—electrical, chemical, civil. However, being a mechanical engineer, the author will use his illustrative examples from this field.

Only a small number of engineers, those who are engaged in research, experimental investigations or advanced design, employ "higher mathematics," under which term advanced algebra and calculus may be included. Even these, when recognizing a difficult mathematical situation can apply to a professional mathematician for help, though they must be sufficiently versed in the usual mathematical language.

The mass of engineers in their everyday activities use only routine arithmetical and algebraic manipulations for the numerical solution of their problems. Most of them would venture the opinion that they have no opportunity to apply their mathematical baggage in their art. They do not realize, however, that they use their knowledge subconsciously. Geometrical concepts and rules are employed by them every day, as well as various theorems of trigonometry. A large number of auxiliary devices and techniques are supplied by mathematicians, such as the slide rule, the alignment chart, the planimeter. True, these are used automatically, but at the beginning the principles of operation of these devices must be understood. A large number of graphical constructions, such as graphostatical analysis of structures, or layouts by means of descriptive geometry are used often. In fact, graphs in cartesian or polar coordinates are plotted on every occasion.

In order to acquire all these techniques, and to employ them intelligently and efficiently, it is essential for the future engineer to have a sound training in high school mathematics, arithmetic, algebra, plane and solid geometry, trigonometry. By this an engineer would not visualize a vast number of special techniques, but rather a thorough knowledge of the basic operations and theorems, accompanied by a fair drill in solution of typical problems.

Deficiency in this preparation precludes the college student from achieving a solid grasp of the elementary calculus in the time allotted to this discipline. He is then handicapped through all his further work in college, when the various engineering professional disciplines are presented in a rather intensive fashion. Whether he wants it or not, the student is forced to make up the deficiency, at least in part, before he is finished with his engineering course. Engineering is a continuous exercise of ingenuity in application of laws of physics, chemistry, mechanics to the various practical problems presented by life. Engineering training consists of demonstrating how these laws are applied. Mathematics, be it in its elementary form, is used commonly as the instrument of demonstration, as a powerful and effective tool in revealing the quantitative relation between the elements involved.

The psychological aspect of the training in elementary mathematics must not be overlooked. It is the business of an engineer to analyze a situation, consciously or subconsciously, before he takes action, in order to segregate the essentials from the incidentals. Solution of problems in arithmetic, algebra and geometry serves very well to form the requisite mental habit. Particularly is it so with those problems which require a certain amount of analysis prior to application of the pertinent manipulations, theorems or formulas.

* An abstract of a paper read before The National Council of Teachers of Mathematics at their summer meeting in New York City on June 27, 1938.

◆ THE ART OF TEACHING ◆

A Device for Teaching the Concept of the Trigonometric Functions

By LAURA BLANK
Hughes High School, Cincinnati, Ohio

PUPILS BEGIN the study of trigonometry, however brief, intensive, or extensive the course, and, whatever the grade in which it is taught, with the definitions of the six trigonometric ratios or functions. Much emphasis is put on these functions. Immediate applications to practical problems are made of the simpler ratios, such as sin 30°, tan 45°, cos 60°, and cot 135°. Shortly, the pupil is introduced to a table of "natural" functions, as we designate them to distinguish them from logarithmic trigonometric functions. The pupil learns to read the tables. He learns to apply them to problems that are not too difficult. It becomes an interesting game to use them. However, in his eagerness to make them serve him in solving problems of many sorts, whether he is a ninth-, tenth-, eleventh-, or twelfth-grade high school pupil, he loses the realization that he is using a ratio in decimal form. Somehow, to him, ratios are common fractions, and common fractions, ratios, but not so decimal fractions. The denominator of the decimal fraction is not apparent to him. Nor is it to us teachers until we make ourselves deliberately conscious of it. In truth, it is for this very reason that we use a decimal fraction, when we prefer it to a common fraction. It is due to this very fact that decimal fractions were invented.

Hence, we as teachers realize that our trigonometry pupils are no longer thinking of trigonometric functions as ratios. We recall to their minds the fundamental definitions and earlier uses of them. But the associations are not there, nor are they easily established. Pupils think of their progress in the subject as the study of two disconnected chapters, truly all pertaining to triangles, but not as a unified field of interest viewed in two ways. $\frac{1}{4} \equiv .25$. If we are buying a quarter of a pound of tea, we use the ratio, $\frac{1}{4}$, in computing; whereas, if we are adding, say 15% and 25% of a certain quantity, we should probably prefer .25 rather than $\frac{1}{4}$.

To overcome this compartmental thinking of trigonometric ratios is the pedagogical problem early in the course. Let each pupil develop empirically an abridged table of sines. He must be led to realize that his table is empirical, not scientific; however, no doubt, it is a development following the chronological unfolding of the science.

The pupil should draw, with a radius of 100 units or 50 units and the lower left-hand corner of a sheet of co-ordinate paper as the center, a quadrant of a circle. Then, with a protractor, using the positive end of the x-axis as the initial side, mark off central angles of 10°, 20°, 30°, 40°, . . . 90°. From the various extremities of the radii which serve as terminal sides of the angles, construct perpendiculars to the common initial side of the angles. All of the constructions should be very carefully made.

When the drawing is complete, read off lengths in units of the respective ordinates of the extremities of the radii which form the terminal sides of the angles. Divide the ordinate of the 10° angle by its terminal side of say 100 or 50 units. The result is sin 10° developed empirically. Do the same for each angle drawn in the figure. The results should be set up in systematic form in several columns with their respective captions: "angle," "ordinate in

units," "terminal side in units," "ratio of ordinate to terminal side," written as a common fraction; finally a column showing this same ratio developed by the pupil in decimal form, to a degree of accuracy consistent with the observed data. A last column might display, for comparison, the values of the respective sines as copied from the printed tables of the textbook, developed by series formulas.

Usually the construction of the sine table is sufficient to clear up and emphasize in the minds of the pupils the concept of trigonometric functions. An occasional class might work out a tangent table as well.

Twelfth year pupils benefit quite as much as younger ones by this development. It fixes for all time the concept of trigonometric ratio.

The problem may be assigned and carefully explained one day. Then two or three days may be given for its assimilation before it need be returned to class. Meanwhile and subsequently, examples in application of the sine function may be worked out, in outside study and in class discussion.

Important Reprint!

THERE is an ample supply of the reprint "The National Council Committee on Arithmetic," by R. L. Morton. This report, published in the October issue of THE MATHEMATICS TEACHER, was reprinted in answer to a demand by arithmetic teachers everywhere. They may be had for 10¢ each, or 5¢ each for quantities of a dozen or more.

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◆ IN OTHER PERIODICALS ◆

By NATHAN LAZAR
Alexander Hamilton High School, Brooklyn, New York

1. Alfred, Brother Bernard, "Another Start." *National Mathematics Magazine*, 13: 133–136. December, 1938.

A presentation of the opening lessons in the calculus. The author disagrees with the procedure described in the March issue of the same magazine by Mr. Edwin Olds in "A Fresh Start."

2. Altwerger, Samuel I. "Mathematics and Science." *High Points*, vol. 20, no. 10, pp. 63–65. October, 1938.

A description of an extra-curricular course given to a small group of students whose intense interest in science, engineering, and mathematics urged them to go beyond the work in the regular high school course. The following are some of the topics treated: empirical formulas, and the formulation of experimental data; the application of vector analysis to concepts in physics such as the triangle law, parallelogram law, resolution of forces, and alternating currents; the application of complex numbers to electricity; the construction and use of nomograms for the solution of quadratic and cubic equations; the mathematical description of constrained motion by means of rectangular, polar, and parametric equations; the elementary notions and techniques of the calculus, and their applications.

It is encouraging to know that there are teachers and pupils who are both competent and willing to carry out the ideal of integrating mathematics with the natural sciences. Teachers of mathematics and science will also be interested in knowing that a secondary school has recently been organized in New York City for those boys and girls who show special aptitude and interest in mathematics and science.

3. Aude, H. T. R., "The Solutions of the Quadratic Equation Obtained by the Aid of Trigonometry." *National Mathematics Magazine*, 13: 118–121. December, 1938.

A clear presentation of a technique not generally known. It should be of special interest to teachers of bright classes and advisers of mathematics clubs.

4. Dockeray, N. R. C., "The Need for Reform in the Teaching of Mathematics." *The Mathematical Gazette*, 22: 321–327. October, 1938.

Writing about the politico-economic conditions of England, the writer says that "we have then every reason to expect a rapid decline in industrial activity starting in the near future. A succession of more or less serious slumps is likely to attend this decline and in consequence a business career will prove continually less and less attractive to the boys who leave our schools. And how is this likely to affect our educational arrangements and, in particular, the teaching of mathematics? Seeing that mathematics was introduced into school teaching as a result of the demand of expanding industrialism and is largely maintained by the exigencies of our commercial civilization, is there not some danger that the impending contraction of industrial activity may go some way toward ousting mathematics from our syllabuses? I myself think that the danger is a grave one."

In order to prevent the above from happening, it is proposed that we continually point out the cultural value of mathematics. "In teaching cultural mathematics, questions involving money or practical matters should be but lightly touched on; emphasis should be laid on the history of mathematical thought, on the part which mathematics has played in the greatest of all civilizations, on the influence which the development of mathematics has had on philosophy and through philosophy on the whole basis of common thought."

5. Hartree, D. R., "The Mechanical Integration of Differential Equations." *The Mathematical Gazette*, 22: 342–364. October, 1938.

A detailed description of the theory and design of mechanical integrators. Fifteen diagrams and twenty references are included.

6. Huntington, Edward V., "The Duplicity of Logic." *Scripta Mathematica*, 5: 149–157, July, 1938.

A very clear treatment of a difficult topic. The duplicity of logic and mathematics consists in their being the official guarantors of truth, and, at the same time, having nothing to do with truth. "We must frankly admit that the field of logic and mathematics does not include the no-

tion of objective truth. But this limitation in the scope of logic and mathematics is not a source of weakness to be deplored; on the contrary, this very limitation is a source of strength, and is to be welcomed as one of the really great achievements of modern thought." The author develops this idea and illustrates its meaning by an apt presentation of some "Pre-mathematical Observations of Space" which he eventually transforms into a mathematical system.

The above article is to be concluded in the next issue.

7. James, Glenn, "The Subject or the Student." *National Mathematics Magazine*, 13: 129–132. December, 1938.

Interesting and keen observations on a familiar topic. The advantages, difficulties, and pitfalls of each procedure are clearly presented.

8. Johnson, Donovan A., "Vitalizing Geometry by the Use of Pictures." *School Science and Mathematics*, 38: 1032–1034. December, 1938.

The author discusses the importance of the use of pictures in geometry and offers an abbreviated list of fourteen topics which can be advantageously illustrated by their use.

9. Kirschner, Rubin, "The Place of General Mathematics in the Junior High School." *High Points*, vol. 20, no. 9, pp. 65–68. November, 1938.

The reasons are given for rejecting the traditional course in algebra for the ninth year. A detailed, week-by-week outline is then included of a course in general mathematics, and its advantages are indicated.

10. McCamey, Kathryn, "Objectives of Ninth-Grade Mathematics in Recent Courses of Study." *School Science and Mathematics*, 38: 972–975. December, 1938.

Resume of a Master of Education thesis at Duke University.

An analysis was made of fifty-three city and state courses of study for ninth-grade mathematics published since 1929 to determine the following:

1. "The type of mathematics provided—whether conventional or reorganized;
2. "How the objectives of traditional and reorganized mathematics differ;
3. "The character of the reorganized courses of study;
4. "Certain other characteristics."

The conclusions the author arrived at are as follows: "Of fifty-three courses of study for ninth grade mathematics published since 1929, two-thirds of them are conventional, and one-third are reorganized mathematics. In nine cases both types of courses were provided for in the manual.

"Half of the courses give general objectives for ninth-grade mathematics, but none tell how those objectives were arrived at.

"Ninth grade mathematics is still predominantly algebra. Even in the reorganized courses, more units in algebra are taught than in arithmetic, geometry, trigonometry, and practical mathematics combined."

11. McCarthy, J. P., "A Difficult Converse." *The Mathematical Gazette*, 22: 365–371. October, 1938.

A good description of the early history and proofs of the perennial theorem: "If the bisectors of the base angles of a triangle are equal, the triangle is isosceles."

The reader who is interested in the history of the problem is referred to an exhaustive article by Dr. J. S. Mackay, in *Proceedings of the Edinburgh Mathematical Society*, vol. 20, 1901–1902.

12. Mossman, Edith L., "The Primary Purpose of Training in Mathematics—Not a Kit of Tools But a Way of Thinking." *School Science and Mathematics*, 38: 992–1002. December, 1938.

A vigorous presentation of an important topic. The author first lists nine reasons why it is not good that a teacher should think of mathematics simply as a tool subject. She then proceeds to careful analysis of some of the outstanding characteristics of the mathematics work in a school system where the primary purpose of that department is believed to be growth in power to think well. In conclusion she indicates the various ways an individual teacher can help and offers a list of twenty-two desirable habits and attitudes that may be formed or strengthened in a mathematics class.

13. Nunn, Sir Percy, "Notes on the Place of Similarity in School Geometry." *The Mathematical Gazette*, 22: 234–249. July, 1938.

According to tradition, parallelism and congruence are considered fundamental concepts of geometry while similarity is treated as a derived one. The author proposes instead that congruence and similarity be considered fundamental and parallelism as a derived idea. He also gives a detailed treatment, with proofs and diagrams, of this novel approach.

The two fundamental axioms of this approach are the following:

1. "The possibility of repeating, anywhere and as often as we please, the construction of a

figure having the same angular and linear dimensions as a given figure.

(2) "The possibility of constructing, anywhere and as often as we please, figures which have the same angular dimensions as a given figure together with linear dimensions which bear a constant ratio to those of the given figure."

The advantages of such an approach are given in some detail.

14. Phillips, E. G., "On Differentiation from First Principles." *The Mathematical Gazette*, 22: 374–376. October, 1938.

Teachers of calculus are acquainted with the difficulty of teaching beginners to differentiate from first principles many of the elementary functions, such as sin x, x^n, log x, and a^x. In this article the author shows that by the use of modern methods of teaching the calculus, the only two elementary functions which have to be differentiated from first principles are x^n, when n is a positive integer, and sin x."

15. Shaw, Allen A., "Note on Roman Numerals." *National Mathematics Magazine*, 13: 126–128. December, 1938.

The author presents many variant forms of numbers written in the Roman System, as well as the rules of the formation of numbers by this system which are not generally well known. These are, in the main, taken from a chapter on Roman numerals published in *The Tutor's Assistant—A Compendium of Arithmetic and a Complete Question Book* by Francis Walkingame, London, 1845. Photostatic reproductions of four pages of that chapter are included.

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NEWS NOTES

Mathematics section meetings were held as a part of the sessions of the Nebraska State Teachers Association October 27–28 in each of the six subdivisions of the state.

District No. 1 at Lincoln

President: Donald Duryea, York. Vice-President: Jack McFarland, Fairbury. "Report of the Hastings Meeting"—Miss Ellen Anderson, Lincoln. "Art in Geometry"—Miss Lorella Ahern, University of Nebraska. "Work of the Joint Commission of Mathematics Association and National Council of Mathematics Teachers on Place of Mathematics in Secondary Schools"—Dr. U. G. Mitchell, University of Kansas.

District No. 2 at Omaha

President: Maybel Burns, Omaha. Secretary: Alma E. Wittenberger, David City. "Report of Nebraska Branch for the National Council."—Miss Wittenberger. "Improving Instruction in Mathematics," Dr. Walter W. Hart, Kenilworth, Illinois.

District No. 3 at Norfolk

Chairman: Maude A. Stewart, West Point. "Report of the Annual Meeting of the Nebraska Division of the National Council of Mathematics Teachers"—L. V. Cross, Norfolk. "A Study of Methods of Teaching as Found in *The Mathematics Teacher* and *School Science and Mathematics*"—Miss Marian Dodderer, Curtis School of Agriculture, Curtis.

District No. 4 at North Platte

President: Wilma Wilson, North Platte. Secretary: L. Ashby Dahl, Cozad. Discussion: "National Council of Mathematics Teachers"—Miss Eva Phalen, Kearney. "Improving Instruction in Mathematics"—Dr. Walter W. Hart, Kenilworth, Illinois.

District No. 5 at McCook

President: George Haskins, Indianola. "Purpose of the Nebraska Association of Mathematics Teachers"—Robert E. Finke, Holdrege. "Preparatory Student"—E. J. Lowry, McCook.

District No. 6 at Chadron

President: Supt. F. H. Holmgrain, Merriman. Secretary: Miss Botsford, Sidney. "Classroom Assignments"—Charles Beers, Rushville. "Report of National Council of Mathematics Teachers"—Miss Lena Meyer, Kearney. "Is Our Mathematics Program Functioning"—Dr. K. O. Broady, University of Nebraska, Lincoln.

The above programs were assembled for publication by Arthur L. Hill, Peru State Teachers College, Peru, Neb.

The Louisiana-Mississippi Sections of the Mathematical Association of America and the National Council of Teachers of Mathematics will meet jointly on March 3–4 at Louisiana State University, University, La. A tentative program is as follows:

Joint preliminary meeting, 2:00 P.M., March 3.

Program of Louisiana-Mississippi section of Mathematical Association of America, 2:30 P.M., March 3.

Joint banquet, University Venetian Room, 7:30 P.M., March 3. Guest speaker will address the group.

National Council meeting, physics lecture room in Nicholson Hall, 8:00 A.M., March 4. Some of the papers which will be presented are: "The Distance of the Stars," Dr. D. V. Guthrie, Physics Dept., L. S. U.; "Mathematics Clubs in High School," Mrs. H. L. Garrett, Istrouma High School; "Visual Aids in the Teaching of Geometry," Miss Jessie Mae Hoag, Jennings High School; "Sidelights in the Teaching of Mathematics," Dr. R. L. O'Quinn, L. S. U.; "The Selective Process in the High School Mathematics Program," Mr. A. C. Maddox, Louisiana State Normal College; "An Analysis of a Freshman Placement Test," Mr. H. T. Karnes, L. S. U.; "The New Organization of Freshman Mathematics Classes at L. S. U.," Major J. P. Cole, L. S. U.

W. H. BRADFORD
Secretary, Mississippi-Louisiana Section
National Council

The Women's Mathematics Club of Chicago and Vicinity held a luncheon meeting on Saturday, November 5. Miss Katherine Aschenbrenner and Miss Marie Echols spoke on the subject, "With Dr. Reeve on the Mathematics Tour through Europe this Past Summer."

On Saturday, December 3, the Women's Mathematics Club of Chicago and Vicinity held a luncheon meeting. Dr. Ruth G. Mason of Wright Junior College spoke on "Mathematical Recreation, or Something Else to Do with Your Leisure Time."

On Saturday, February 4, occurred the annual dinner given by the Men's and Women's Mathematics Clubs of Chicago and Vicinity. The dinner will be at the Chicago Woman's Club, and the speaker will be Professor Arthur Haas of the Department of Physics of Notre Dame University.

LENORE JOHN

The Association of Mathematics Chairmen of New York City and the Association of Teachers of Mathematics of New York City announce a Joint Annual Luncheon, preceded by a Panel Discussion, on Saturday, March 11, at the Hotel Astor. The principal speaker will be Professor Oswald Veblen, of the Institute of Advanced Studies at Princeton University. His subject will be: "A Modern Approach to the Elementary Euclidean Geometry."

The committee consists of S. Welkowitz, Franklin K. Lane High School, and Dr. Nathan Lazar, Alexander Hamilton High School, discussion; Dr. S. I. Greitzer, Benjamin Franklin High School, arrangements; E. Schacker, Benjamin Franklin High School, publicity; and S. H. Barkan, Benjamin Franklin High School, chairman.

The program will be of interest to teachers of junior and senior high schools, evening schools, and to all concerned with the teaching of mathematics.

The David Eugene Smith Club of Teachers College, Columbia University, has carried on an active season with the leadership of the following officers: President, I. S. Turner, of Sydney, Australia; Vice-President, Louise Baurli of Brooklyn, N.Y.; Secretary-Treasurer, Homer Howard, of Gainesville, Florida.

At the club's first meeting in October, Professor W. D. Reeve entertained the group with motion pictures in both colored and uncolored films, showing scenes in Germany where he conducted a group of American students studying education during the past summer.

Dr. Nathan Lazar, of the Alexander Hamilton High School, Brooklyn, and author of *Logic in Geometry*, was the club's speaker at its November meeting. In his discussion of the teaching of algebra, Dr. Lazar demonstrated a method he has developed as a result of his experience with less capable pupils. He employs three practical questions in connection with all algebraic operations, namely: (1) What? (description), (2) What for? (purpose), and (3) By what right? (justification). This device lifts algebra out of the realm of blind manipulation, Dr. Lazar said.

Professor George Mullins of Barnard College, Columbia University, addressed the December meeting on "Graphical Solutions of Equations" and illustrated his talk with lantern slides.

On the evening of January 9, the club gave a dinner in honor of Professor C. B. Upton, of Teachers College, who will be on sabbatical leave during the spring session of the current year.

The Mathematics Section of the New York Society for the Experimental Study of Education met for dinner at the Men's Faculty Club of Columbia University on January 14.

Dr. W. L. Schaaf, of the Brooklyn College Department of Education, spoke after dinner on "The Teaching of the History of Mathematics." Dr. Schaaf discussed his subject from two aspects: (1) the possible values in studying the history of mathematics, and (2) how history materials may be worked into the grade placement chart outline of the course tentatively proposed by the Joint Commission.

The values which may most probably be derived from the study of history of mathematics were stated by the speaker as follows: (1) to appreciate the cultural significance of mathematics, (2) to clarify important concepts, and (3) to arouse curiosity and stimulate interest.

Skillful, psychologically adroit presentation is necessary to make this material workable in the course. A logical chronological presentation of history of mathematics as such is not practicable in high school, Dr. Schaaf said.

Annual Luncheon

Those who wish to attend the Luncheon Meeting for Delegates and Representatives on February 25, at noon, in the Hotel Carter, Cleveland, should send reservations to Mrs. Florence Brooks Miller, Chairman of State Representatives, 3295 Avalon Road, Shaker Heights, Ohio. This luncheon is part of the program of the Twentieth Annual Meeting of the National Council of Teachers of Mathematics, which will be held in Cleveland, Ohio, February 24 and 25. Price of the luncheon $1.

Important Reprint!

There is an ample supply of the reprint "The National Council Committee on Arithmetic," by R. L. Morton. This report, published in the October issue of THE MATHEMATICS TEACHER, was reprinted in answer to a demand by arithmetic teachers everywhere. They may be had for 10¢ each, or 5¢ each for quantities of a dozen or more.